Homework 8: Other coordinates

This homework is due Friday, 9/26 rsp Tuesday 9/30.

1 Change the following points to cylindrical coordinates: a) $(2\sqrt{3}, 2, 7)$
b) $(4, -3, 6)$
Change to spherical coordinates: c) $(0, \sqrt{3}, 1)$ d) $(-1, 1, \sqrt{6})$

2 a) Identity the surface $r^2 = (z - 5)^2$.
b) Identify the surface given in spherical coordinates as $\rho \sin(\phi) = 2$.

3 a) Identify the surface $2r^2 - z^2 = 1$.
b) Identify the surface $\cos^2(\phi) = \sin^2(\phi)$.

4 a) Identity the surface whose equation is given in cylindrical coordinates as

$$\cos(\theta) + \sin(\theta) = 1/r \ .$$

b) Identify the surface whose equation is given in spherical coordinates as

$$\rho^2(\sin^2(\phi)\sin^2(\theta) + \cos^2(\phi)) = 16 \ .$$

5 a) Use the Mathematica command "RevolutionPlot3D" to plot the surface which is given by

$$z = (\sin(3\theta) + \cos(3\theta))e^{-r^2} \ .$$

(It is used like Plot3D but now r and θ are the variables.)
b) Use the Mathematica command "SphericalPlot3D" to plot a bumpy sphere, where the radius ρ depends on the angles ϕ and θ.

$$\rho(\phi, \theta) = (3 + \cos(\theta) + \sin(11\theta)\sin(13\phi)) \ .$$

(Also this is used like Plot3D but now ϕ and θ are the variables.)

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RevolutionPlot3D [ f [ r , t ] ,  { r ,0 ,3 } ,{ t ,0 ,2 Pi }]
SphericalPlot3D [ f [ s , t ] ,  { s ,0 , Pi } ,{ t ,0 ,2 Pi }]
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Main definitions

A point (x, y) in the plane has the **polar coordinates** $r = \sqrt{x^2 + y^2} \geq 0, \theta = \text{arctg}(y/x)$. We have the relation $(x, y) = (r\cos(\theta), r\sin(\theta))$. We can chose the arctan values in $(-\pi/2, \pi/2]$ and add π if $x < 0$ or $x = 0, y < 0$. A point (x, y, z) in space has the **cylindrical coordinates** (r, θ, z), where (r, θ) are the polar coordinates of (x, y).

A curve given in polar coordinates as $r(\theta) = f(\theta)$ is called a **polar curve**. It can in Cartesian coordinates be described as $\vec{r}(t) = \langle f(t)\cos(t), f(t)\sin(t) \rangle$.

Spherical coordinates use the distance ρ to the origin as well as two angles θ and ϕ. The first angle θ is the polar angle in polar coordinates of the xy coordinates and ϕ is the angle between the vector $\vec{OP}$ and the z-axis. The relation is $(x, y, z) = (\rho\cos(\theta)\sin(\phi), \rho\sin(\theta)\sin(\phi), \rho\cos(\phi))$. We have

$$
\begin{aligned}
x &= \rho\cos(\theta)\sin(\phi), \\
y &= \rho\sin(\theta)\sin(\phi), \\
z &= \rho\cos(\phi)
\end{aligned}
$$