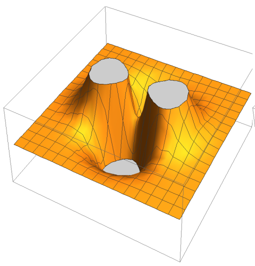
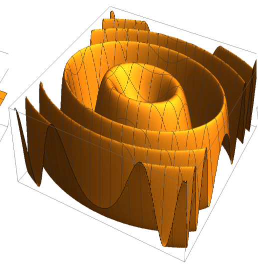
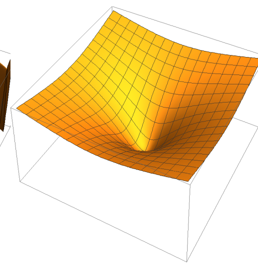
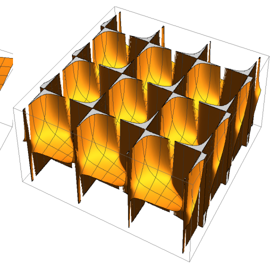
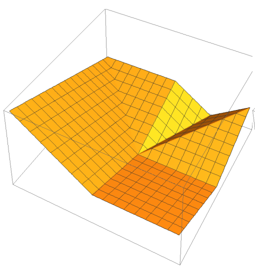
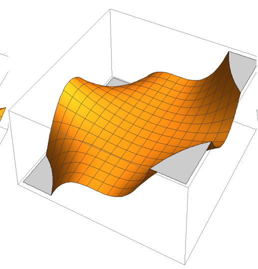
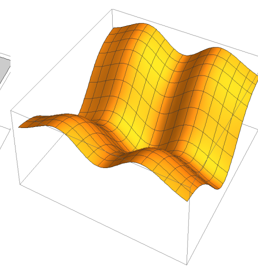
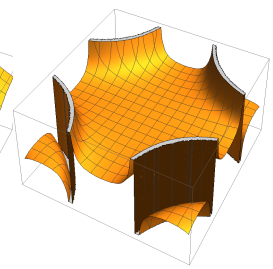
Homework 5: Functions of 2 and 3 variables

This homework is due Friday, 9/19 rsp Tuesday 9/23.

1 Match the following graphs with the functions $f(x,y)$.

I II III IV

V VI VII VIII

$f(x,y) =$	I,II,III,IV,V,VI,VII,VIII
$\lvert x - \lvert y - \lvert x\rvert\rvert\rvert$	
$x^2 + x^3y^2$	
$\exp(-x^2-y^2)(x^2-y^2)$	
$\sin(x^2+2y^2)$	
$\log(x^2+y^2+1)$	
$\exp(-x^2)x^2 - \exp(-y^2)$	
$\sec(xy)$	
$\tan(x)/\tan(y)$	

2 a) Plot the **Monkey saddle**, the graph of $z = x^3 - 3xy^2$. What are the traces of the graph, the intersection of the graph with the xy-plane?

b) Find the domain and range of the function $f(x,y) = \sqrt{x^2 - y^2 + 1}$ and plot the graph of the function over the domain, where it is defined

3 a) Plot the graph and contour map of the function $f(x,y) = \frac{x-y}{x^2+y^2}$
b) Plot the graph and contour map of the function $f(x,y) = \frac{(x/y)}{x^2+y^2}$.
You can explore this with technology.

4 Find an equation for the surface consisting of all points P for which the distance from P to the x-axes is twice the distance from P to the yz-plane. Identify the surface.

5 a) Draw the surface $4y^2 + z^2 - x - 16y - 4z + 20 = 0$.
b) Draw the surface $x - z^2 + y^2 + 4y = 1$.

Main definitions

The **domain** D of a function $f(x,y)$ is the set of points where f is defined, the range is $\{f(x,y) \mid (x,y) \in D\}$. The **graph** of $f(x,y)$ is the surface $\{(x,y,f(x,y)) \mid (x,y) \in D\}$ in space. The set $f(x,y) = c =$ const is **contour curve** or **level curve** of f. The collection of all contour curves $\{f(x,y) = c\}$ is called the **contour map** of f. A function of three variables $g(x,y,z)$ assigns to three variables x,y,z a real number $g(x,y,z)$. We can visualize it by **contour surfaces** $g(x,y,z) = c$, where c is constant. Important are **traces**, the intersections of the surfaces with the coordinate planes.

The elliptic paraboloid $z - x^2 - y^2 = 0$ and hyperboloid $z - x^2 + y^2 = 0$ are examples of graphs $z - f(x,y) = 0$. The one sheeted hyperboloid $x^2 + y^2 - z^2 = 1$ and two sheeted hyperboloid $x^2 + y^2 - z^2 = -1$ or cylinder $x^2 + y^2 = 1$ are examples of surfaces of revolution $x^2 + y^2 - g(z) = 0$.