Homework 4: Lines and Planes

This homework is due Wednesday, 9/17 rsp Thursday 9/18.

1 a) Find a parametrization for the line L through $P = (1, 2, 3)$ perpendicular to the plane $2x - y + 3z = 7$.
b) Intersect the line L with the plane $x + y + z = 12$.

2 a) Parametrize the three line segments of the triangle $A \to B \to C$, where $A = (1, 1, 1)$, $B = (2, 3, 4)$ and $C = (4, 5, 6)$.
b) Find the equation $ax + by + cz = d$ of the plane through A, B, C.

3 a) Find an equation of the plane that passes through the line of intersection of the planes $x - z = 1$ and $y + z = 3$ and is perpendicular to the plane $x + y - 2z = 1$.
b) Find the distance of the plane found in a) to the origin $(0, 0, 0)$.

4 a) Parametrize the line L through $P = (2, 1, 2)$ that intersects the line $x = 1 + t, y = 1 - t, z = 2t$ perpendicularly.
b) What is the distance from this line L to the origin $(0, 0, 0)$?

5 a) Find the two intersection points of the cylinder $x^2 + z^2 = 1$ with the line $\vec{r}(t) = \langle 10 + 2t, 3t, 4t + 2 \rangle$.
b) Find the distance of the line to the y axes, the center of the cylinder.

Main definitions

A point $P = (p, q, r)$ and a vector $\vec{v} = \langle a, b, c \rangle$ define the **line**

$$L = \{ \langle x, y, z \rangle = \langle p, q, r \rangle + t \langle a, b, c \rangle, t \in \mathbf{R} \} .$$

Solving for t gives the **symmetric equations**

$$\frac{x - p}{a} = \frac{y - q}{b} = \frac{z - r}{c} .$$

A point P and two vectors $\vec{v}, \vec{w}$ define a **plane** $\Sigma = \{ \vec{OP} + t\vec{v} + s\vec{w}$, where t, s are real numbers $\}$.
An example is $\Sigma = \{ \langle x, y, z \rangle = \langle 1, 1, 2 \rangle + t \langle 2, 4, 6 \rangle + s \langle 1, 0, -1 \rangle \}$. This is called the **parametric description** of a plane.

The implicit equation of the plane $\vec{x} = \vec{x}_0 + t\vec{v} + s\vec{w}$ is

$$ax + by + cz = d ,$$

where $\langle a, b, c \rangle = \vec{v} \times \vec{w}$ is a vector normal to the plane and d is obtained by plugging in $\vec{x}_0$.