Homework 32: Flux integral

This homework is due Monday, 11/24 rsp Tuesday 11/25 just before Thanksgiving. If no orientation is given, the orientation is assumed to be "outwards".

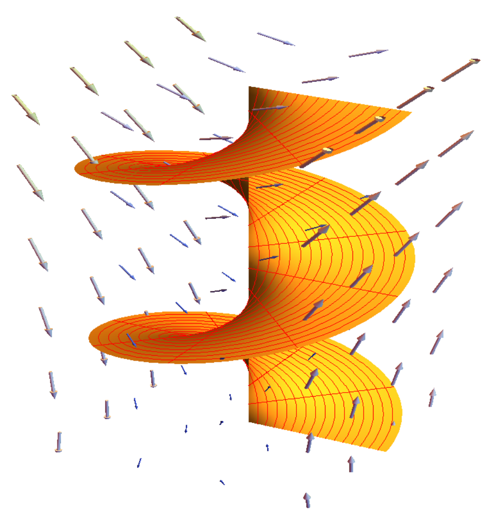
1 Evaluate the flux integral $\int\int_S \vec{F} \cdot d\vec{S}$ if

$$\vec{F}(x, y, z) = \langle 3z, 3y, 3x \rangle \ ,$$

and S is the helicoid

$$\vec{r}(u, v) = \langle u \cos v, u \sin v, v \rangle, 0 \leq u \leq 1, 0 \leq v \leq 4\pi$$

which has an upward orientation.



2 Evaluate the flux integral $\int\int_S \vec{F} \cdot d\vec{S}$ for the vector field

$$\vec{F}(x, y, z) = \langle xz, x, y \rangle \ ,$$

where S is the hemisphere $x^2 + y^2 + z^2 = 25, x \geq 0$, oriented in the direction of the negative x-axes.

3 Evaluate the flux integral $\int\int_S \vec{F} \cdot d\vec{S}$ for the vector field

$$\vec{F}(x, y, z) = \langle x, y, 5 \rangle \ ,$$

where S is the boundary of the region enclosed by the cylinder $x^2 + z^2 = 1$ and the planes $y = 0$ and $x + y = 2$.

4 The temperature $f(x,y,z)$ at a point (x,y,z) is equal to the distance from the center $(0,0,0)$. Find the flux of the heat flow field $\vec{F} = -\nabla f$ across a sphere S of radius 2 centered at $(0,0,0)$.

5 Let $\vec{F}(x,y,z)$ be an inverse square field, that is

$$\vec{F}(x,y,z) = c\langle x,y,z\rangle/\rho^3$$

with $\rho = \sqrt{x^2+y^2+z^2}$. Show that the flux of $\vec{F}$ across a sphere S with center at the origin and radius R is independent of the radius of S.

Main points

If a surface S is parametrized as $\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$ over a domain G in the uv-plane and $\vec{F}$ is a vector field, then the **flux integral** of $\vec{F}$ through S is

$$\int\int_G \vec{F}(\vec{r}(u,v)) \cdot (\vec{r}_u \times \vec{r}_v)\ dudv\ .$$

With the short hand notation $d\vec{S} = (\vec{r}_u \times \vec{r}_v)\ dudv$ representing an infinitesimal normal vector to the surface, this can be written as $\int\int_S \vec{F} \cdot d\vec{S}$. The interpretation is that if $\vec{F} =$ fluid velocity field, then $\int\int_S \vec{F} \cdot d\vec{S}$ is the amount of fluid passing through S in unit time.