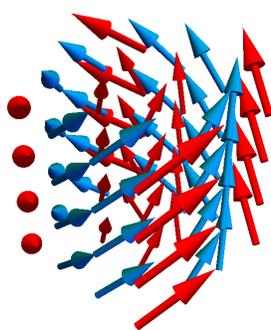## Homework 27: Vector fields

This homework is due Wednesday, 11/12 rsp Thursday 11/13.
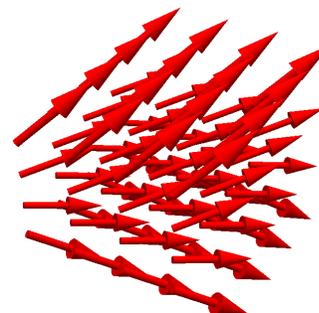
1 Match the vector fields $\vec{F}$ with the plots labeled A-D.
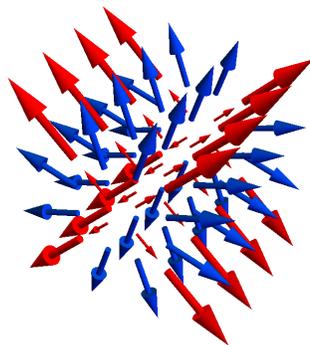a) $\vec{F}(x, y, z) = \langle -y, x, 1 \rangle$, b) $\vec{F}(x, y, z) = \langle 1, 1, z \rangle$
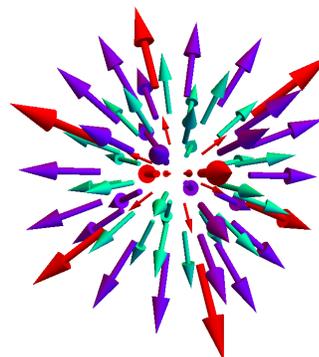c) $\vec{F}(x, y, z) = \langle x, 0, z \rangle$, d) $\vec{F}(x, y, z) = \langle x, y, z \rangle$

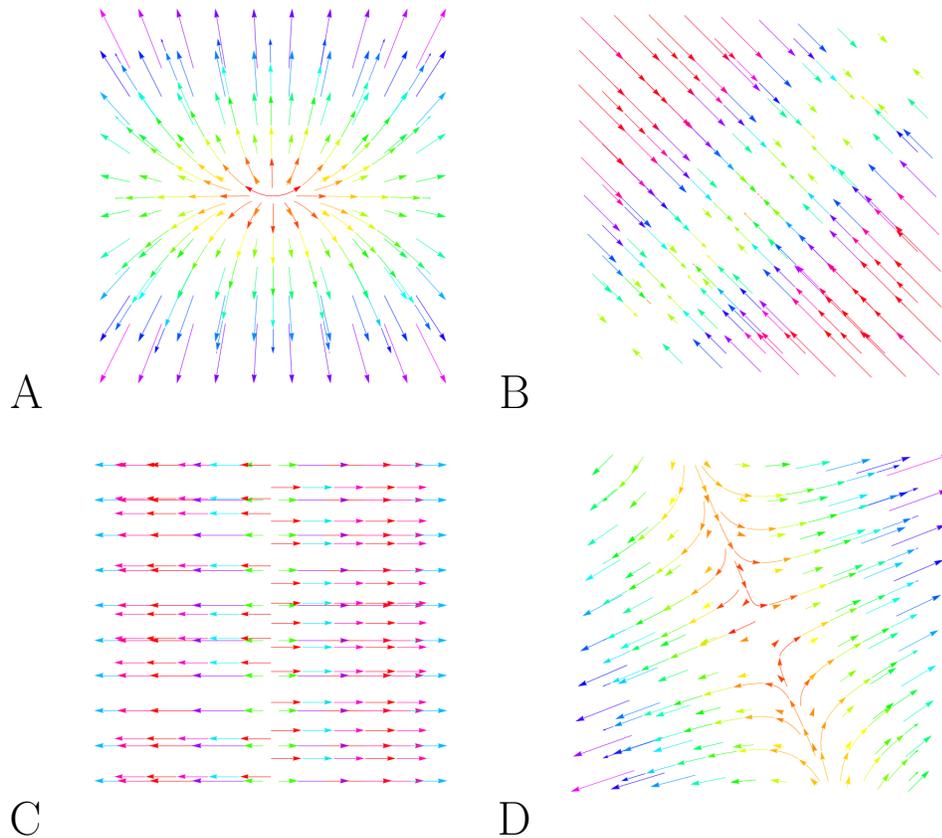A   B

C   D

2 a) Find the gradient vector field of f, where $f(x, y) = \tan(7x - 5y)$.
b) Given the vector field $\vec{F} = \langle P, Q \rangle = \langle \frac{x}{\sqrt{x^2 - 3y^2}} + y, x - \frac{3y}{\sqrt{x^2 - 3y^2}} \rangle$.
Check that $Q_x - P_y = 0$ and find a function $f(x, y)$ for which $\nabla f = \vec{F}$.

3 Match the functions f with the plots of their gradient fields labeled $A - D$. Give reasons for your choices.

a) $f(x, y) = x^2 + 2y^2$, b) $f(x, y) = x(x + y)$
c) $f(x, y) = \cos(x - y)$, d) $f(x, y) = \sin(\sqrt{1 + x^2})$

A

B

C

D

4 a) Sketch the vector field $\vec{F}(x, y) = \langle 2, x \rangle$ and then sketch some flow lines. What shape to these flow lines appear to have?

b) Now find the flow line $\vec{r}(t)$ with $\vec{r}(0) = \langle 0, 0 \rangle$.

5 a) Let $\vec{F}(x, y) = \langle x, y \rangle (r^2 - 2r)$ with $r = |\langle x, y \rangle|$. Plot the vector field using Mathematica.

b) Make a stream plot of the field $\vec{F}(x, y) = \langle x^2 y, x + y \rangle$ using Mathematica. Example code is below.

```
VectorPlot[{x+y,x^2},{x,1,2},{y,-1,1}]
StreamPlot[{x+y,x^2},{x,1,2},{y,-1,1}]
VectorPlot3D[{x,y,z},{x,1,2},{y,1,2},{z,1,2}]
```