

Homework 23: Surface area

This homework is due Monday, 11/3 resp Tuesday 11/4.

- 1 Find the surface area of the surface given by

$$z = \frac{2}{3}(x^{3/2} + y^{3/2}), \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1.$$

- 2 Find the area of the surface given by the **helicoid**

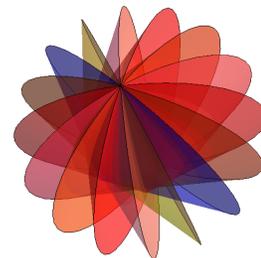
$$\vec{r}(u, v) = \langle u \cos(v), u \sin(v), v \rangle.$$

with $0 \leq u \leq 1$, $0 \leq v \leq \pi$.

A decorative paper lantern is made of 8 surfaces. Each is parametrized by

3
$$\vec{r}(t, z) = \langle 10z \cos(t), 10z \sin(t), z \rangle$$

with $0 \leq t \leq 2\pi$ and $0 \leq z \leq 1$ and then translated or rotated. Find the total surface area of the lantern.

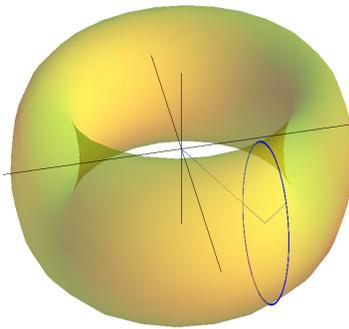


- 4 The figure shows the torus obtained by rotating about the z -axis the circle in the xz -plane with center $(b, 0, 0)$ and radius $a < b$. Parametric equations for the torus are

$$\begin{aligned} x &= b \cos \theta + a \cos \alpha \cos \theta \\ y &= b \sin \theta + a \cos \alpha \sin \theta \\ z &= a \sin \alpha, \end{aligned}$$

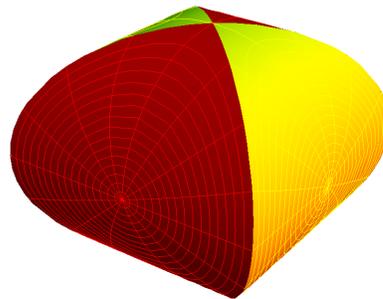
where θ and α are the angles shown in the figure. Find the surface

area of the torus.



- 5 The volume and surface area of the solid obtained by intersecting the solid cylinder $y^2 + z^2 \leq 1$ with the solid cylinder $x^2 + z^2 \leq 1$ has been found by Archimedes already. Find the surface area of the

surface S bounding this solid.



Main definitions:

A surface $\vec{r}(u, v)$ parametrized on a parameter domain R has the **surface area**

$$\int \int_R |\vec{r}_u(u, v) \times \vec{r}_v(u, v)| \, du \, dv .$$

Examples:

$\vec{r}(u, v)$	$ \vec{r}_u \times \vec{r}_v $
$\langle \rho \cos(u) \sin(v), \rho \sin(u) \sin(v), \rho \cos(v) \rangle$	$\rho^2 \sin(v) $
$\langle u, v, f(u, v) \rangle$	$\sqrt{1 + f_u^2 + f_v^2}$
$\langle f(v) \cos(u), f(v) \sin(u), v \rangle$	$f(v) \sqrt{1 + f'(v)^2}$