

Homework 21: Double integrals

This homework is due Wednesday, 10/29 resp Thursday 10/30.

- 1 a) (4 points) Find the iterated integral

$$\int_0^1 \int_0^2 3xy/\sqrt{x^2 + 2y^2} dy dx .$$

- b) (4 points) Now compute

$$\int_0^1 \int_0^2 3xy/\sqrt{x^2 + 2y^2} dx dy .$$

- c) (2 points) Wouldn't Fubini assure that a) and b) are the same? What change would be needed in b) to make the results agree?

- 2 Calculate the double integral

$$\iint_R \frac{2x}{1 + xy} dA$$

over the region $R = \{(x, y) \mid 0 \leq y \leq x, 0 \leq x \leq 1\}$.

- 3 a) Evaluate the double integral

$$\int_0^2 \int_y^{2y} xy dx dy .$$

- b) Evaluate the integral of xy over the same region but as a type I integral

$$\int_0^2 \int_{\dots}^{\dots} xy dy dx + \int_2^4 \int_{\dots}^{\dots} xy dy dx .$$

- 4 We want to find the volume of the region below the graph of $f(x, y) = xy^2$, where R is the region enclosed by the curves $x = 0$, $x = \sqrt{1 - y^2}$. In other words, find $\iint_R xy^2 dA$.

- 5 Evaluate the integral:

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx dy .$$

Main definitions

If R is a two dimensional region and $f(x, y)$ a function of two variables, the **double integral** $\iint_R f(x, y) dA$ is defined the limit of the Riemann sum $(1/n^2) \sum_{(i/n, j/n) \in R} f(i/n, j/n)$ for $n \rightarrow \infty$. Depending on how we order the sum we write $dA = dx dy$ or $dA = dy dx$ and reduce to single variable integrals.

A **type I region** is of the form

$$R = \{(x, y) \mid a \leq x \leq b, c(x) \leq y \leq d(x)\} .$$

An integral over such a region is called a **type I integral**

$$\iint_R f dA = \int_a^b \int_{c(x)}^{d(x)} f(x, y) dy dx .$$

A **type II region** is of the form

$$R = \{(x, y) \mid c \leq y \leq d, a(y) \leq x \leq b(y)\} .$$

An integral over such a region is called a **type II integral**

$$\iint_R f dA = \int_c^d \int_{a(y)}^{b(y)} f(x, y) dx dy .$$

Fubini's theorem allows to switch the order of integration over a rectangle, if the function f is continuous:

$$\int_a^b \int_c^d f(x, y) dx dy = \int_c^d \int_a^b f(x, y) dy dx .$$