Homework 20: Global extrema

This homework is due Monday, 10/27 rsp Tuesday 10/28.

1 Find the extreme values of f on the region described by the inequality.

$$f(x, y) = 2x^2 + 3y^2 - 4x - 5, \quad x^2 + y^2 \leq 16$$

2 We suppose that the Cobb Douglas production formula $Q(L, K) = bL^{1/4}K^{3/4} = 100$, which tells that the quantity Q is constant. What values of L and K minimizes the cost function $C(L, K) = 4L + 5K$?

3 Find the absolute maximum and minimum values of

$$f(x, y) = e^{-x^2 - y^2}(x^2 + 2y^2);$$

on the disk $D = \{x^2 + y^2 \leq 4\}$.

4 Use Lagrange multipliers to find the maximum and minimum values of f subject to the given constraints.

$$f(x, y) = \frac{1}{x} + \frac{1}{y}; \quad \frac{1}{x^2} + \frac{1}{y^2} = 1 \ .$$

5 A package in the shape of a rectangular box can be mailed by the US Postal Service if the sum of its length and girth (the perimeter of a cross-section perpendicular to the length) is at most 108. Find the dimensions of the package with largest volume $V(x, y, z) = xyz$ that can be mailed under the constraint $x + 2y + 2z \leq 108$.

Standard assumption is still that all functions have continuous first and second derivatives.

A point (x_0, y_0) is an **absolute maximum** = **global maximum** on a domain R, if $f(x, y) \leq f(x_0, y_0)$ for all (x, y) in R.

To find a global maximum, we look at the local maxima and minima as well as the maxima and minima on the boundary. The later is a Lagrange problem. If the domain is unbounded, we also have to look at the behavior of the function when $(x, y) \to \infty$.

Example.
$f(x, y) = x^2 + y^2 - x^4 - y^4$ has a local minimum at $(0, 0)$ but this is not a global minimum because $f(1000, 1000)$ for example is smaller than $f(0, 0) = 0$.
If $f(x, y)$ is considered on the domain $R = \{x^2 + y^2 \leq 1\}$ then the situation has changed and we need to look at extrema on the boundary too. One can see that $(0, 0), (\pm 1, 0), (0, \pm 1)$ are all global minima.

A region R is bounded if there is r such that R is contained in a disc of radius r. A region is closed if it contains all boundary points. A continuous function on a region which is bounded and closed always has a global maximum and a global minimum on R.