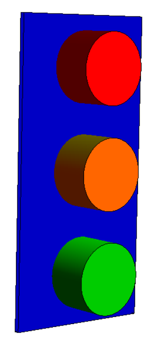
Homework 19: Lagrange multipliers

This homework is due Friday, 10/24 rsp Tuesday 10/28.

1 Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint(s).

$$f(x,y) = 3e^{xy}; \quad x^3 + y^3 = 16 \,.$$

2 The material to build a traffic light is $g(x,y) = 6 + 6\pi xy + 3\pi x^2 = 12$ is fixed (the radius of each cylinder is x and the height is y and the constant 6 is the material for the back plate). We want to build a light for which the shaded region with volume $f(x,y) = 3\pi x^2 y$ is maximal. Use the Lagrange method.



3 Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint(s).

$$f(x,y,z) = 3x - y - 3z; \quad x + y - z = 0, \; x^2 + 2z^2 = 1$$

4 Use Lagrange multipliers to prove that the triangle with maximum area that has a give perimeter p is equilateral. *Hint:* Use Heron's formula for the area:

$$A = \sqrt{s(s-x)(s-y)(s-z)}$$

where $s = p/2$ and x, y, z are the lengths of the sides.

5 a) The plane $4x - 3y + 8z = 5$ intersects the cone $z^2 = x^2 + y^2$ in an ellipse. Use Lagrange multipliers to find the highest and lowest points on the ellipse.
b) Find the points on the surface $y^2 = 9 + xz$ that are closest to the origin.

Hi Avery, I only now realize how bad the solutions were from last year! In HW 19, Problem 5a and 5b were bad. You might have seen that but I wanted to alert you. Below is an attempt to make it better. Oliver

Main definitions

The system of equations $\nabla f(x,y) = \lambda \nabla g(x,y), g(x,y) = 0$ for the three unknowns x, y, λ are called **Lagrange equations**. The variable λ is a **Lagrange multiplier**.

$\nabla f(x,y,z) = \lambda \nabla g(x,y,z), g(x,y,z) = 0$ are the **Lagrange equations** in three dimensions.

$\nabla f(x,y,z) = \lambda \nabla g(x,y,z) + \mu \nabla h(x,y,z), g(x,y,z) = 0, h(x,y,z) = 0$ are the **Lagrange equations** in three dimensions with two constraints. There are two Lagrange multipliers λ, μ.

Lagrange theorem: Extrema of f on the constraint $g = c$ are either solutions of the Lagrange equations or critical points of g.