Homework 18: Extrema

This homework is due Wednesday, 10/22 rsp Thursday 10/23.

1 Find the local maximum and minimum values of the function.

$$f(x, y) = 5xy + \frac{5}{x} + \frac{5}{y}$$

2 Classify the critical points of the function

$$f(x, y) = 7e^{2y}(4y^2 - x^2) \ .$$

3 Find the local maximum and minimum values and saddle point(s) of the function.

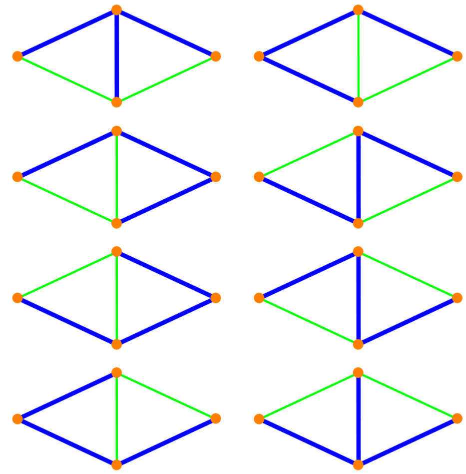
$$f(x, y) = 2 \sin x \sin y, -\pi < x < \pi, -\pi < y < \pi$$

4 Companies like **Netflix** or **Hulu** track movie preferences. One can visualize preferences on parameter spaces which is the **intelligence - emotion** plane. Based on viewing habits, the service decides what you want to see. Your profile is a function $f(x, y)$. Maximizing this function allows the company to pick movies for you. Assume that your user profile is the function $f(x, y) = -2x^3 + 9x^2 - 12x - y^2$. Find and classify all the critical points and especially find the local maxima of f.

5 Graph theorists look at the **Tutte polynomial** $f(x,y)$ of a network. We work with the Tutte polynomial

$$f(x,y) = x + 2x^2 + x^3 + y + 2xy + y^2$$

of the **Kite network**. Classify the two critical using the second derivative test.



Remark. The polynomial is useful: $xf(1-x,0)$ tells in how many ways one can color the nodes of the network with x colors and $f(1,1)$ tells how many spanning trees there are. This picture illustrates that the number of spanning trees of the kite graph is $f(1,1) = 8$ as you see the 8 possible trees.

Main definitions

Standard assumption is always that functions are smooth in the sense that all first and second partial derivatives are continuous.

A point (x_0, y_0) is a **critical point** of f if $\nabla f(x_0, y_0) = \langle 0, 0 \rangle$.

Fermat's theorem: if f has a local maximum or local minimum at (x_0, y_0) then (x_0, y_0) is a critical point

Second derivative test: Assume (x_0, y_0) is a critical point. If $D < 0$ then it is a saddle point. If $D > 0, f_{xx} < 0$ then (x_0, y_0) is a local maximum. If $D > 0, f_{xx} > 0$ then (x_0, y_0) is a local minimum.