Homework 17: Directional Derivatives

This homework is due Monday, 10/20 rsp Tuesday 10/21.

1 Find the gradient of

$$f(x, y, z) = \sqrt{x + yz}\ .$$

at the point $P = (1, 3, 1)$ and use it to find the rate of change of f at P in the direction of the vector $\vec{u} = \langle 2/7, 3/7, 6/7 \rangle$.

2 a) Find the directional derivative of the function $f(x, y) = \log(x^2 + y^2)$ at the point $P = (2, 1)$ in the direction of the vector $\vec{v} = \langle -1, 2 \rangle$. (We use the notation $\log = \ln$). b) Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at the point $P = (1, -1, 3)$ in the direction from P to $Q = (2, 4, 5)$.

3 a) Find the direction in which the rate of change of $f(x, y, z) = \frac{(x+y)}{z}$ is maximal at the point $P = (1, 1, -1)$.
b) Find the maximal rate of change at $(1, 1, -1)$ in that direction found in a).

4 Find the directions in which the directional derivative of $f(x, y) = ye^{-xy}$ at the point $(0, 2)$ has the value 1.

5 [Arlington-Belmont-Waltham-Cambridge]
On http://goo.gl/fhY1rl,
you find a map of some suburbs of Boston (an original copy in office 432).
a) The map contains some creeks. Find an example which confirms the rule that water crosses level curves perpendicularly.
b) The map shows some railway tracks. Check whether the rule applies that railway trays follow the level curves of the height.
c) Estimate the maximal directional derivative on this map. How would you measure this maximal steepness?

Main definitions:

If f is a function of several variables and $\vec{v}$ is a unit vector then $D_{\vec{v}} f = \nabla f \cdot \vec{v}$ is called the **directional derivative** of f in the direction $\vec{v}$.

If $\vec{v} = \nabla f / |\nabla f|$, then the directional derivative is

$$D_v f = \nabla f \cdot \nabla f / |\nabla f| = |\nabla f| \ .$$

This means f **increases**, if we move into the direction of the gradient. The length of the gradient vector $|\nabla f|$ is the slope in the gradient direction.