Homework 1: Geometry and Distance

This homework is due Wednesday, 9/10 respectively Thursday 9/11. Remember we have a strict HW policy. HW is due at the beginning of each class. No mailbox submissions.

1 If a is the distance of the point $(2, 3, -10)$ to the yz plane and b is the distance of $(2, 3, -10)$ to the x axes, find $a^2 + b^2$. Can you see what the answer for a general point (x_0, y_0, z_0) is?

2 a) Find its center and radius of the sphere:

$$x^2 + y^2 + z^2 - 40x - 8y - 2z - 18 = 0 \ .$$

b) Find the distance of the sphere in a) to the unit sphere $x^2 + y^2 + z^2 = 1$.

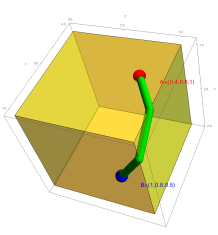
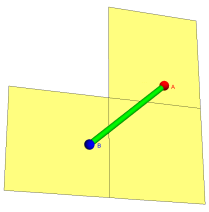
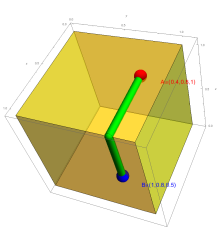
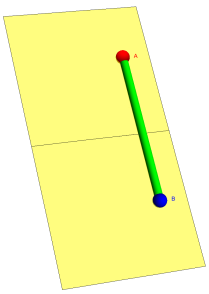
3 a) Find an equation of the largest sphere with center $(6, 9, 8)$ that is contained in the first octant.

b) Find the equation for the sphere centered at $(6, 10, 8)$ which passes through the origin $(0, 0, 0)$.

4 a) Draw and describe in words the surface $(y-1)^2+(z+2)^2 = 16$ in three dimensional space R^3.

b) Draw and describe in words the surface $y^2 = z^2$ in three dimensional space R^3.

5 An ant moves on the unit cube bound by the walls $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ from the point $A = (0.4, 0.8, 1)$ to the point $B = (1, 0.8, 0.5)$. Compute the length of the two obvious paths, where one passes over three faces, the other only over two. Which one is shorter?

Main definitions

A point in the **plane** has **coordinates** $P = (x, y)$. A point in **space** has coordinates $P = (x, y, z)$. The coordinate signs define 4 **quadrants** in the plane and 8 **octants** in space. These regions by intersect at the **origin** $O = (0, 0)$ or $O = (0, 0, 0)$, and are separated by **coordinate planes** $\{x = 0\}, \{y = 0\}, \{z = 0\}$ which intersect in **coordinate axes** like the z-axes $\{y = 0, x = 0\}$.

The **Euclidean distance** between two points $P = (x, y, z)$ and $Q = (a, b, c)$ in space is defined as $d(P, Q) = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$. The distance between a point P and a geometric object S like a line or plane or sphere is the minimal distance $d(P, Q)$ which can be achieved among all points Q located on S.

A **circle** of radius r centered at $P = (a, b)$ is the collection of points in the plane which have distance r from P. A **sphere** of radius ρ centered at $P = (a, b, c)$ is the collection of points in space which have distance ρ from P. The equation of a sphere is $(x-a)^2 + (y-b)^2 + (z-c)^2 = \rho^2$.

We **complete the square** of $x^2 + bx + c = 0$ by adding$(b/2)^2 - c$ on both sides to get $(x+b/2)^2 = (b/2)^2 - c$. Solving for x gives $x = -b/2 \pm \sqrt{(b/2)^2 - c}$. **Example:** Find the center and radius of the circle $x^2+8x+y^2 = 9$. **Solution:** Add 16 on both sides to get $x^2+8x+16+y^2 = 25$ which is $(x+4)^2+y^2 = 25$, a circle of radius $r = 5$ centered at $(-4, 0)$.