Name:

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MWF 11 Gijs Heuts
MWF 11 Siu-Cheong Lau
MWF 12 Erick Knight
MWF 12 Kate Penner
TTH 10 Peter Smillie
TTH 10 Jeff Kuan
TTH 10 Yi Xie
TTH 11:30 Jeff Kuan
TTH 11:30 Jameel Al-Aidroos

- Start by printing your name in the above box and **check your section** in the box to the left.

- Do not detach pages from this exam packet or unstaple the packet.

- Please write neatly. Answers which are illegible for the grader cannot be given credit.

- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.

- All functions can be differentiated arbitrarily often unless otherwise specified.

- No notes, books, calculators, computers, or other electronic aids can be allowed.

- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) [T] [F] The surface described in spherical coordinates as $\phi = \pi/4$ is the xy plane.

2) [T] [F] The length of the unit tangent vector $\vec{T}$ for a curve $\vec{r}(t)$ is independent of t.

3) [T] [F] For all vectors $\vec{v}$ and $\vec{w}$ the vector $\vec{w} \times (\vec{w} \times \vec{v})$ is perpendicular to $\vec{v}$.

4) [T] [F] There is a point (x, y, z) in space, for which the cylindrical coordinates (r, θ, z) and spherical coorindates (ρ, θ, ϕ) satisfy $(r, \theta, z) = (\rho, \theta, \phi - \pi/2)$.

5) [T] [F] The two planes $x + y - z = 1$ and $-x - y + z = 2$ intersect in a line.

6) [T] [F] $(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v}) = 0$ implies $|u| = |v|$.

7) [T] [F] The contour curves $\sin(x) + y = 1$ and $\sin(x) + y = 2$ do not intersect.

8) [T] [F] There is a vector $\vec{v}$ for which the vector projection $\text{proj}_{\vec{v}}(\vec{j})$ is equal to $2\vec{j}$.

9) [T] [F] $(\vec{k} \times \vec{i}) \times \vec{i} = \vec{j} \times (\vec{i} \times \vec{k})$

10) [T] [F] If a curve $\vec{r}(t)$ lies in a plane, goes through the point $(1, 1, 1)$, and has the binormal vector $\vec{B}(t) = \langle 3, 4, 5 \rangle$, then the plane is $3x + 4y + 5z = 12$.

11) [T] [F] The angle between $\vec{r}\,'(t)$ and $\vec{r}\,''(t)$ is always 90 degrees.

12) [T] [F] A line intersects a hyperbolic paraboloid always in 2 distinct points.

13) [T] [F] There is a quadric surface, each of whose intersections with the coordinate planes is either an ellipse or a parabola.

14) [T] [F] The equation $x^2 - y^2 - z^2 = 1$ defines a one-sheeted hyperboloid.

15) [T] [F] The function $f(x, y) = 1/(1 + x^2 + y^2)$ is continuous everywhere.

16) [T] [F] If the number $\vec{u} \cdot (\vec{v} \times \vec{w})$ is positive, then $(\vec{w} \times \vec{v}) \cdot \vec{u}$ is positive.

17) [T] [F] The number $|\vec{u} \times (\vec{v} \times \vec{w})|$ is the volume of the parallelepiped spanned by $\vec{u}, \vec{v}$ and $\vec{w}$.

18) [T] [F] The set of points P for which the distance of P to the point $(0, 0, 0)$ is 1 less than the distance to the point $(0, 0, 2)$ is a paraboloid.

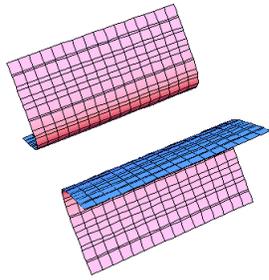19) [T] [F] If $\vec{v}, \vec{w}$ are two nonzero vectors, then the projection vector $\text{proj}_{\vec{w}}(\vec{v})$ can be longer than $\vec{v}$.
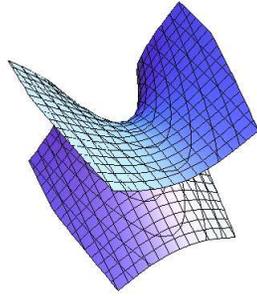
20) [T] [F] The number $|\vec{v} \times \vec{w}|$ is the area of the parallelogram spanned by $\vec{v}$ and $\vec{w}$.
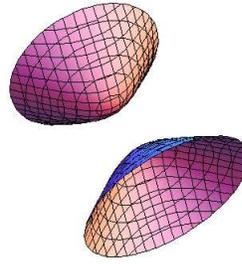
Problem 2a) (5 points)

Match the equations with the pictures. No justifications are necessary in this problem.
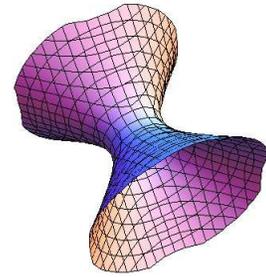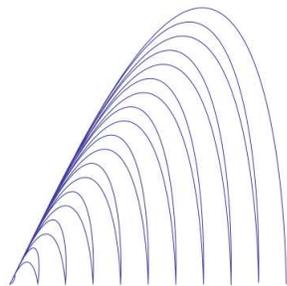

I II III IV

Enter I,II,III,IV here	Equation
	$x^2 + y - z^2 - 1 = 0$
	$y^2 - 2z^2 - 1 = 0$
	$x^2 - y^2 + z^2 + 1 = 0$
	$x^2 - y^2 + z^2 - 1 = 0$
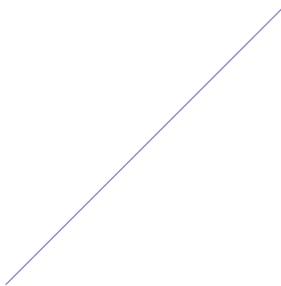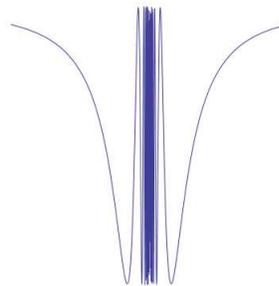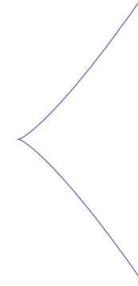
1 2 3 4

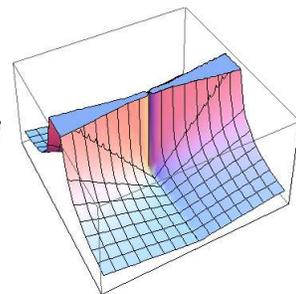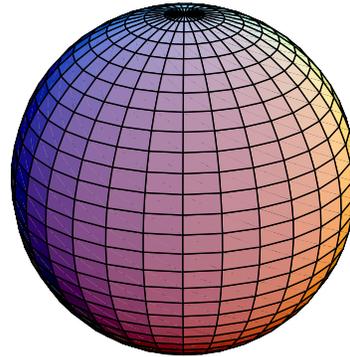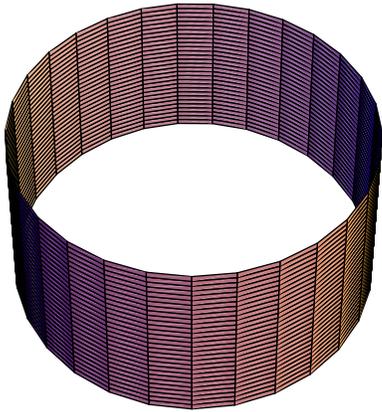| Enter 1,2,3,4 here	Equation
	$\langle t^4, 1 + t^5 \rangle$
	$\langle t\cos(5t), t\cos(5t) \rangle$
	$\langle |t\cos(5t)|, |t\sin(10t)| \rangle$
	$\langle 3 + 2t, \cos(1/t) \rangle$
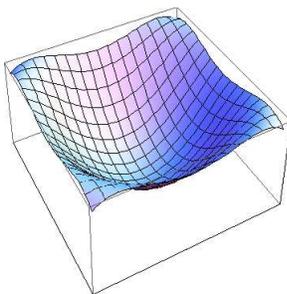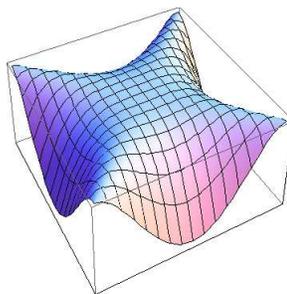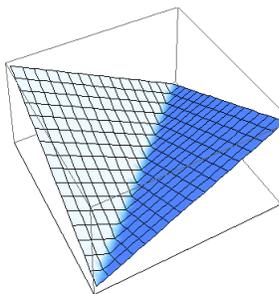
A B C D

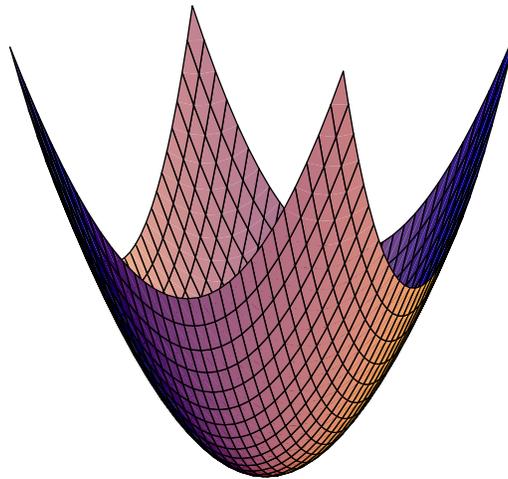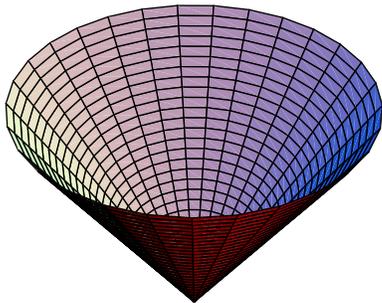| Enter A,B,C,D here	Equation
	$f(x, y) = |x/y|$
	$f(x, y) = \sin(x^2 + y^2)$
	$f(x, y) = |x - y|$
	$f(x, y) = \cos(x^2 - y^2)$

Match the surfaces with their parametrizations as well as with the description either in cylindrical coordinates (r, θ, z) or in spherical coordinates (ρ, ϕ, θ).

I

II

III

IV

Enter I,II,III,IV here	Parametrization of the surface
	$\langle 3\cos(\theta), 3\sin(\theta), 2z \rangle$.
	$\langle x, y, 3x^2 + 3y^2 \rangle$
	$\langle 3\sin(\phi)\cos(\theta), 3\sin(\phi)\sin(\theta), 3\cos(\phi) \rangle$
	$\langle 3z\cos(\theta), 3z\sin(\theta), 2z \rangle$
Enter I,II,III,IV here	Description in cylindrical or spherical coordinates
	$z = 3r^2$
	$3z = 2r$
	$\rho = 3$
	$r = 3$

A tetrahedron has the vertices $A = (1, 1, 0), B = (3, 2, 0), C = (2, 1, 1), D = (3, 2, 1)$ with base triangle A, B, C.

a) (5 points) Find the height of the tetrahedron.

b) (5 points) The volume of a tetrahedron is the base area times height divided by 3. What is the volume of the tetrahedron with vertices A,B,C,D.

Find the symmetric equation
$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$
of the intersection of the two planes

$$2x + y + z = 4$$

and

$$x - y + 2z = 5 .$$

What is the distance between the two cylinders $x^2 + y^2 = 1$ and $(z - 2)^2 + (x - 5)^2 = 4$?

Find the arc length of the parameterized curve

$$\vec{r}(t) = \langle 2 \sin(t), \frac{t^4}{4} + \frac{1}{2t^2}, 2 \cos(t) \rangle$$

from $t = 1$ to $t = 2$.
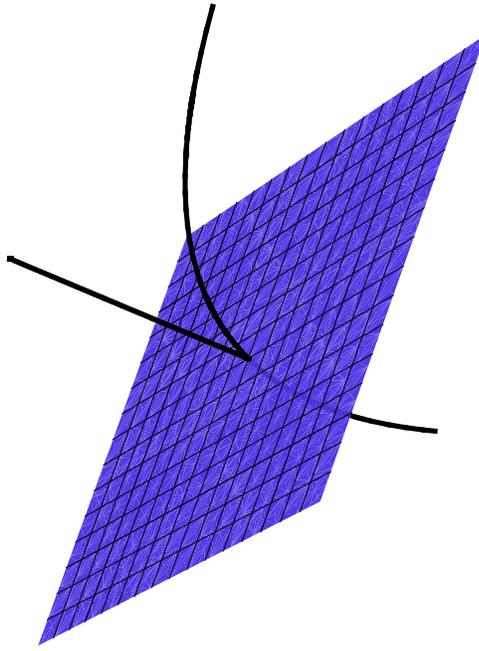
Problem 7) (10 points)

At time $t = 0$ two trapeze artists have positions $\vec{r}(0) = \langle 0, 0, 25 \rangle$ and $\vec{s}(0) = \langle 10, 0, 23 \rangle$ and velocities $\vec{r}\,'(0) = \langle 2, 0, 1 \rangle$ and $\vec{s}\,'(0) = \langle -3, 0, 2 \rangle$. They both experience a constant gravitational acceleration $\langle 0, 0, -10 \rangle$. Find the paths $\vec{r}(t), \vec{s}(t)$ and determine at which point the artists meet.

Problem 8) (10 points)

The angle between a curve and a plane is defined as $\pi/2 - \alpha$, where α is the angle between the normal vector to the plane and the velocity vector of the curve at the point of intersection.

a) (3 points) Find a normal vector to the plane $x + y - z/2 = 1$.

b) (3 points) What is the velocity vector to the curve $C : \vec{r}(t) = \langle 1, 0, 0 \rangle + t\langle 1, 1, 3 \rangle + t^2 \langle 1, 1, 1 \rangle$ at time $t = 0$?

c) (4 points) Find the angle (in radians) between the plane $x + y - z/2 = 1$ and the curve C at the point of intersection $\vec{r}(0)$.

The intersection of the paraboloid

$$x^2 + y^2 - z = 5$$

with the plane

$$x + y = 5$$

is a curve. Find the parametrization of this curve.

a) (3 points) Parametrize the plane containing the three points $A = (1, 1, 1), B = (1, 3, 2)$ and $C = (3, 4, 5)$.

b) (4 points) Parametrize the sphere which is centered at $(1, 1, 1)$ and has radius 3.

c) (3 points) Parametrize the surface which is given in spherical coordinates as $\rho = 3 + \sin(\phi) \sin(\theta)$.