Name:

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|---|
| MWF 9 Oliver Knill |
| MWF 9 Chao Li |
| MWF 10 Gijs Heuts |
| MWF 10 Yu-Wen Hsu |
| MWF 10 Yong-Suk Moon |
| MWF 11 Rosalie Belanger-Rioux |
| MWF 11 Gijs Heuts |
| MWF 11 Siu-Cheong Lau |
| MWF 12 Erick Knight |
| MWF 12 Kate Penner |
| TTH 10 Peter Smillie |
| TTH 10 Jeff Kuan |
| TTH 10 Yi Xie |
| TTH 11:30 Jeff Kuan |
| TTH 11:30 Jameel Al-Aidroos |

- Start by printing your name in the above box and **check your section** in the box to the left.

- Do not detach pages from this exam packet or unstaple the packet.

- Please write neatly. Answers which are illegible for the grader cannot be given credit.

- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.

- All functions can be differentiated arbitrarily often unless otherwise specified.

- No notes, books, calculators, computers, or other electronic aids can be allowed.

- You have 90 minutes time to complete your work.

| 1 | | 20 |
|---|---|---|
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 10 |
| 6 | | 10 |
| 7 | | 10 |
| 8 | | 10 |
| 9 | | 10 |
| 10 | | 10 |
| Total: | | 110 |

Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F — The point $(x, y, z) = (-1, -1, -1)$ is in spherical coordinate described as $(\rho, \theta, \phi) = (\sqrt{3}, 5\pi, 3\pi/4)$

2) T F — If $|\vec{v} \times \vec{w}| = 0$ then $\vec{v} = \vec{0}$ or $\vec{w} = \vec{0}$.

3) T F — The surface $z^2 + 4y^2 = x^2 + 1$ is a two sheeted hyperboloid.

4) T F — The surface $4x^2 - 4x + y^2 - 2y - 120 = -z^2$ is an ellipsoid.

5) T F — The parametrized lines $\vec{u}(t) = \langle 1 + 2t, 2 - 5t, 1 + t \rangle$ and $\vec{v}(t) = \langle 3 - 4t, -3 + 10t, 2 - 2t \rangle$ are the same line.

6) T F — The surface $\sin(x) = z$ contains lines which are parallel to the y-axis.

7) T F — If $\vec{u} \cdot \vec{v} = 0$, $\vec{v} \cdot \vec{w} = 0$ and $\vec{v}$ is not the zero vector, then $\vec{u} \cdot \vec{w} = 0$.

8) T F — The curvature of a curve depends upon the speed at which one travels upon it.

9) T F — Two lines in space that do not intersect must be parallel.

10) T F — A line in space can intersect an elliptic paraboloid in 4 points.

11) T F — If $\vec{u} \times \vec{v} = 0$ and $\vec{u} \cdot \vec{v} = 0$, then one of the vectors $\vec{u}$ and $\vec{v}$ is zero.

12) T F — If the velocity vector $\vec{r}\,'(t)$ and the acceleration vector $\vec{r}\,''(t)$ of a curve are parallel at time $t = 1$, then the curvature $\kappa(t)$ of the curve is zero at time $t = 1$.

13) T F — If the speed of a parametrized curve is constant over time, then the curvature of the curve $\vec{r}(t)$ is zero.

14) T F — The length of the vector projection of a vector $\vec{v}$ onto a vector $\vec{w}$ is always equal to the length of the vector projection of $\vec{w}$ onto $\vec{v}$.

15) T F — A quadric $ax^2 + by^2 + cz^2 = 1$ is contained in the interior of a sphere $x^2 + y^2 + z^2 < 100$, then the constants $a, b, c$ are all positive and the quadric is an ellipsoid.

16) T F — There is a hyperboloid of the form $ax^2 + by^2 - cz^2 = 1$ which has a trace which is a parabola.

17) T F — The set of points in space which have distance 1 from the line $x = y = z$ form a cylinder.

18) T F — The velocity vector of a parametric curve $\vec{r}(t)$ always has constant length.

19) T F — The volume of a parallelepiped spanned by $\vec{u}, \vec{v}, \vec{w}$ is $|(\vec{u} \times \vec{v}) \times \vec{w}|$.

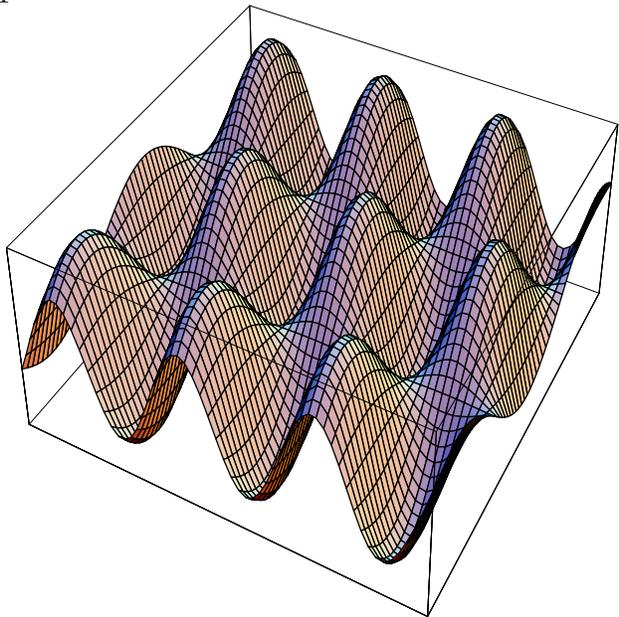20) T F — The equation $x^2 + y^2/4 = 1$ in space describes an ellipsoid.

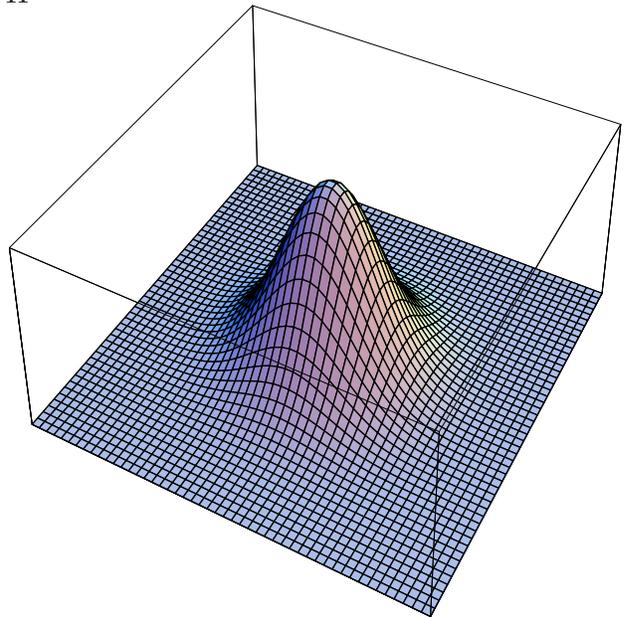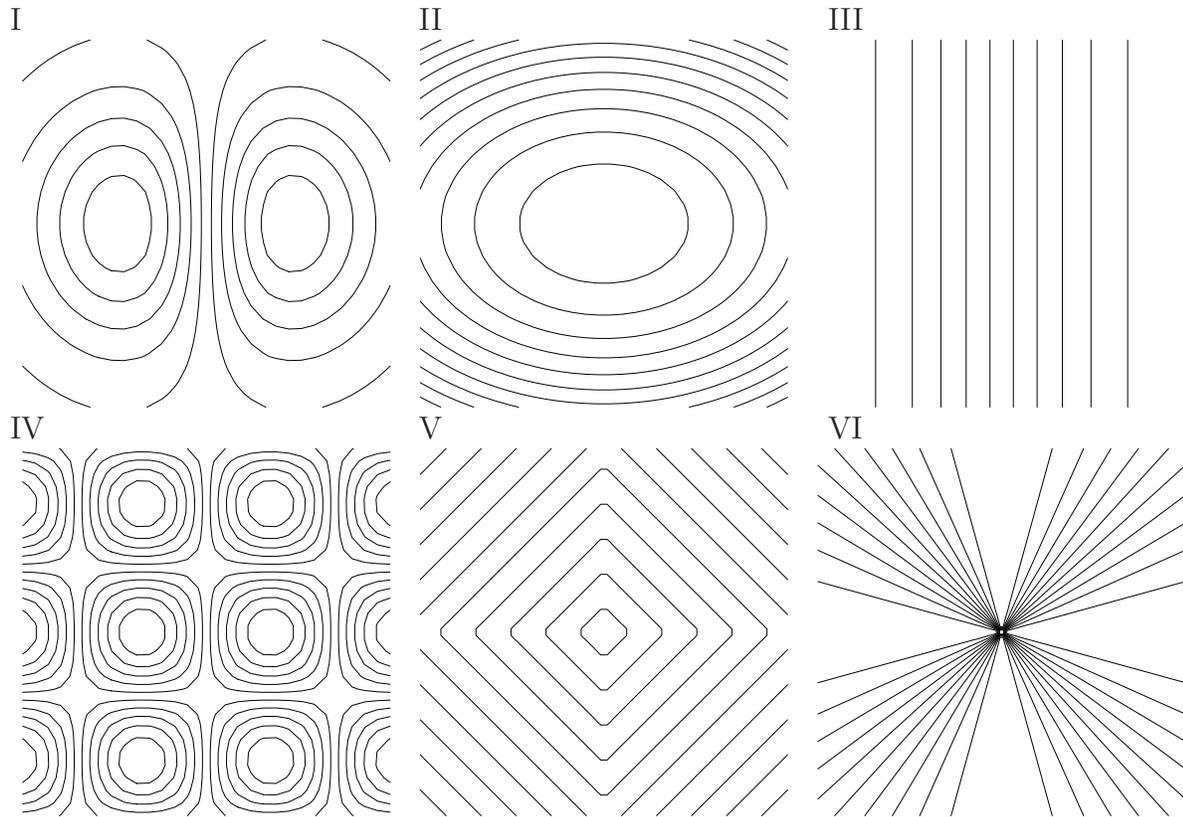Match the equation with their graphs. No justifications are needed.
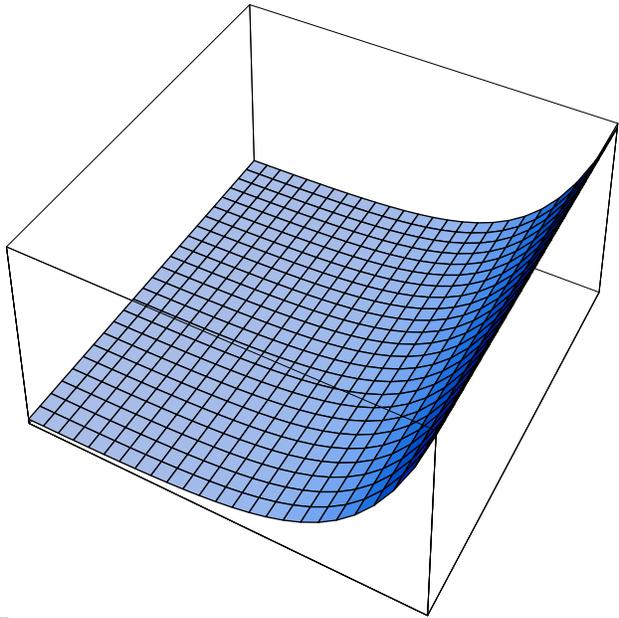


I
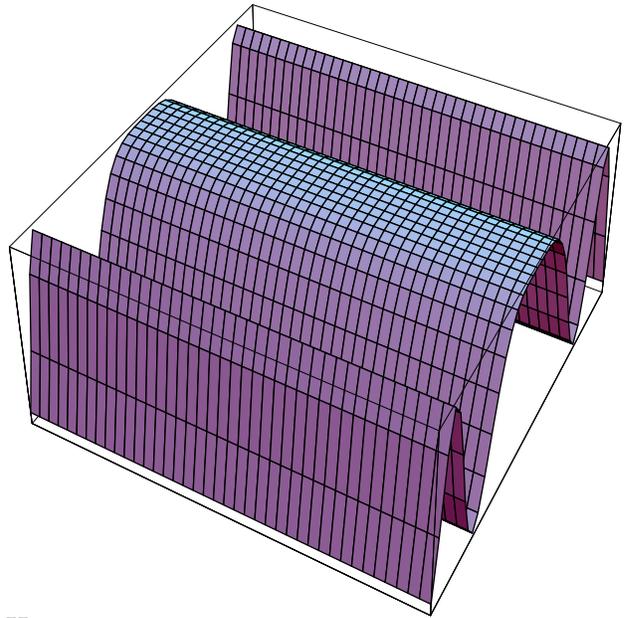


II



III



IV

| Enter I,II,III,IV here | Equation |
| --- | --- |
| | $z = \sin(5x)\cos(2y)$ |
| | $z = \cos(y^2)$ |
| | $z = e^{-x^2-y^2}$ |
| | $z = e^x$ |

Problem 2b) (4 points)

Match the contour maps with the corresponding functions $f(x, y)$ of two variables. No justifications are needed.

I

II

III

IV

V

VI

| Enter I,II,III,IV,V or VI here | Function $f(x, y)$ |
|---|---|
| | $f(x, y) = \sin(x)$ |
| | $f(x, y) = x^2 + 2y^2$ |
| | $f(x, y) = |x| + |y|$ |
| | $f(x, y) = \sin(x)\cos(y)$ |
| | $f(x, y) = xe^{-x^2-y^2}$ |
| | $f(x, y) = x^2/(x^2 + y^2)$ |

Match the following points in cartesian coordinates with the points in spherical coordinates:

a) $(x, y, z) = (\sqrt{2}, 0, 0)$
b) $(x, y, z) = (0, \sqrt{2}, 0)$
c) $(x, y, z) = (0, 0, \sqrt{2})$
d) $(x, y, z) = (1, 1, 0)$
e) $(x, y, z) = (1, 0, 1)$
f) $(x, y, z) = (0, 1, 1)$

1) $(\rho, \phi, \theta) = (\sqrt{2}, 0, 0)$.
2) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, \pi/4)$.
3) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, 0)$.
4) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, \pi/2)$.
5) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/4, \pi/2)$.
6) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/4, 0)$.

| Problem 3) (10 points) |

a) (7 points) Find a parametric equation for the line which is the intersection of the two planes $2x - y + 3z = 9$ and $x + 2y + 3z = -7$.

b) (3 points) Find a plane perpendicular to both planes given in a) which has the additional property that it passes through the point $P = (1, 1, 1)$.
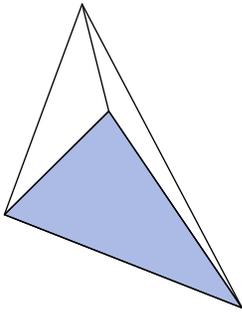
| Problem 4) (10 points) |

Given the vectors $\vec{v} = \langle 1, 1, 0 \rangle$ and $\vec{w} = \langle 0, 0, 1 \rangle$ and the point $P = (2, 4, -2)$. Let $\Sigma$ be the plane which goes through the origin $(0, 0, 0)$ and which contains the vectors $\vec{v}$ and $\vec{w}$. Let $S$ be the unit sphere $x^2 + y^2 + z^2 = 1$.

a) (6 points) Compute the distance from $P$ to the plane $\Sigma$.

b) (4 points) Find the shortest distance from $P$ to the sphere $S$.

| Problem 5) (10 points) |

a) (6 points) Find an equation for the plane through the points $A = (0, 1, 0)$, $B = (1, 2, 1)$ and $C = (2, 4, 5)$.

b) (4 points) Given an additional point $P = (-1, 2, 3)$, what is the volume of the tetrahedron which has $A, B, C, P$ among its vertices.

**A useful fact which you can use without justification in b):** the volume of the tetrahedron is $1/6$ of the volume of the parallelepiped which has AB, AC, and AP among its edges.

6

**Problem 6) (10 points)**

The parametrized curve $\vec{u}(t) = <t, t^2, t^3>$ (known as the "twisted cubic") intersects the parametrized line $\vec{v}(s) = \langle 1 + 3s, 1 - s, 1 + 2s \rangle$ at a point $P$. Find the angle of intersection.
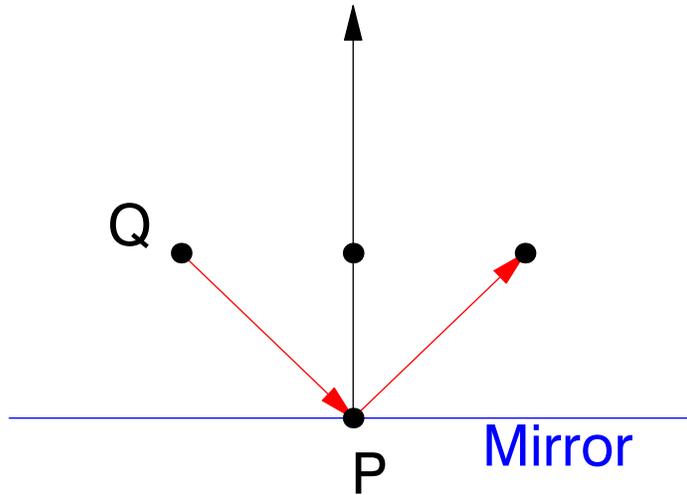
**Problem 7) (10 points)**

Let $\vec{r}(t)$ be the space curve $\vec{r}(t) = (\log(t), 2t, t^2)$, where $\log(t)$ is the natural logarithm (denoted by $\ln(t)$ in some textbooks).

a) What is the velocity and what is the acceleration at time $t = 1$?

b) Find the length of the curve from $t = 1$ to $t = 2$.

**Problem 8) (10 points)**

A planar mirror in space contains the point $P = (4, 1, 5)$ and is perpendicular to the vector $\vec{n} = \langle 1, 2, -3 \rangle$. The light ray $\vec{QP} = \vec{v} = \langle -3, 1, -2 \rangle$ with source $Q = (7, 0, 7)$ hits the mirror plane at the point $P$.

a) (4 points) Compute the projection $\vec{u} = \vec{P}_{\vec{n}}(\vec{v})$ of $\vec{v}$ onto $\vec{n}$.

b) (6 points) Identify $\vec{u}$ in the figure and use it to find a vector parallel to the reflected ray.

Q

Mirror

P

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**Problem 9) (10 points)**

We know the acceleration $\vec{r}\,''(t) = \langle 2, 1, 3 \rangle + t\langle 1, -1, 1 \rangle$ and the initial position $\vec{r}(0) = \langle 0, 0, 0 \rangle$ and initial velocity $\vec{r}\,'(0) = \langle 11, 7, 0 \rangle$ of an unknown curve $\vec{r}(t)$. Find $\vec{r}(6)$.

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**Problem 10) (10 points)**

Intersecting the elliptic cylinder $x^2 + y^2/4 = 1$ with the plane $z = \sqrt{3}x$ gives a curve in space.

a) (3 points) Find the parametrization of the curve.

b) (3 points) Compute the unit tangent vector $\vec{T}$ to the curve at the point $(0, 2, 0)$.

c) (4 points) Write down the arc length integral and evaluate the arc length of the curve.