

LAGRANGE



GAUSS



STOKES



AMPERE



Math

21a

Fall

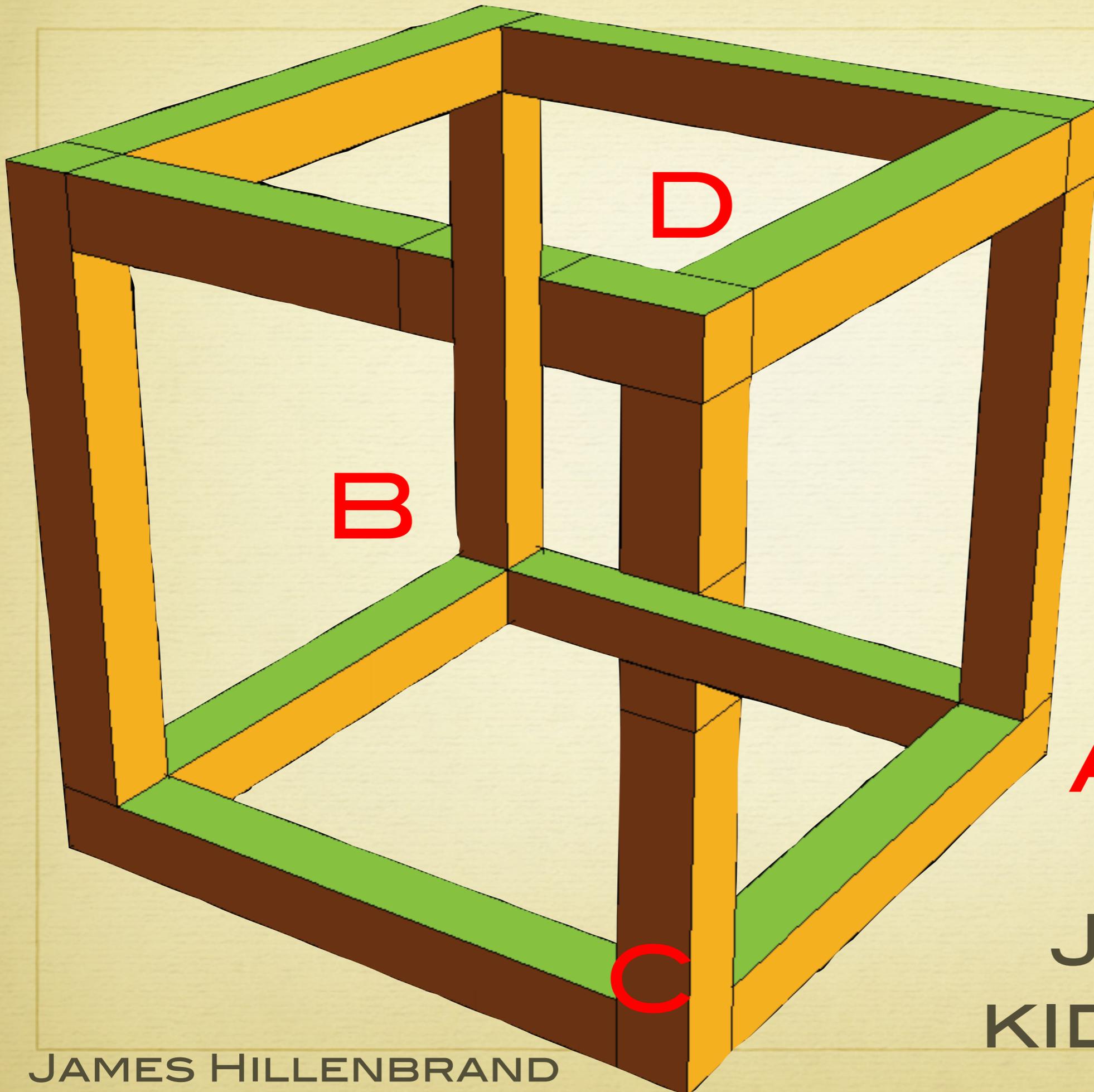
2013

OLIVER

KNILL

12/10/13

Problem



FIND THE
DISTANCE
BETWEEN AB
AND CD

A

JUST
KIDDING

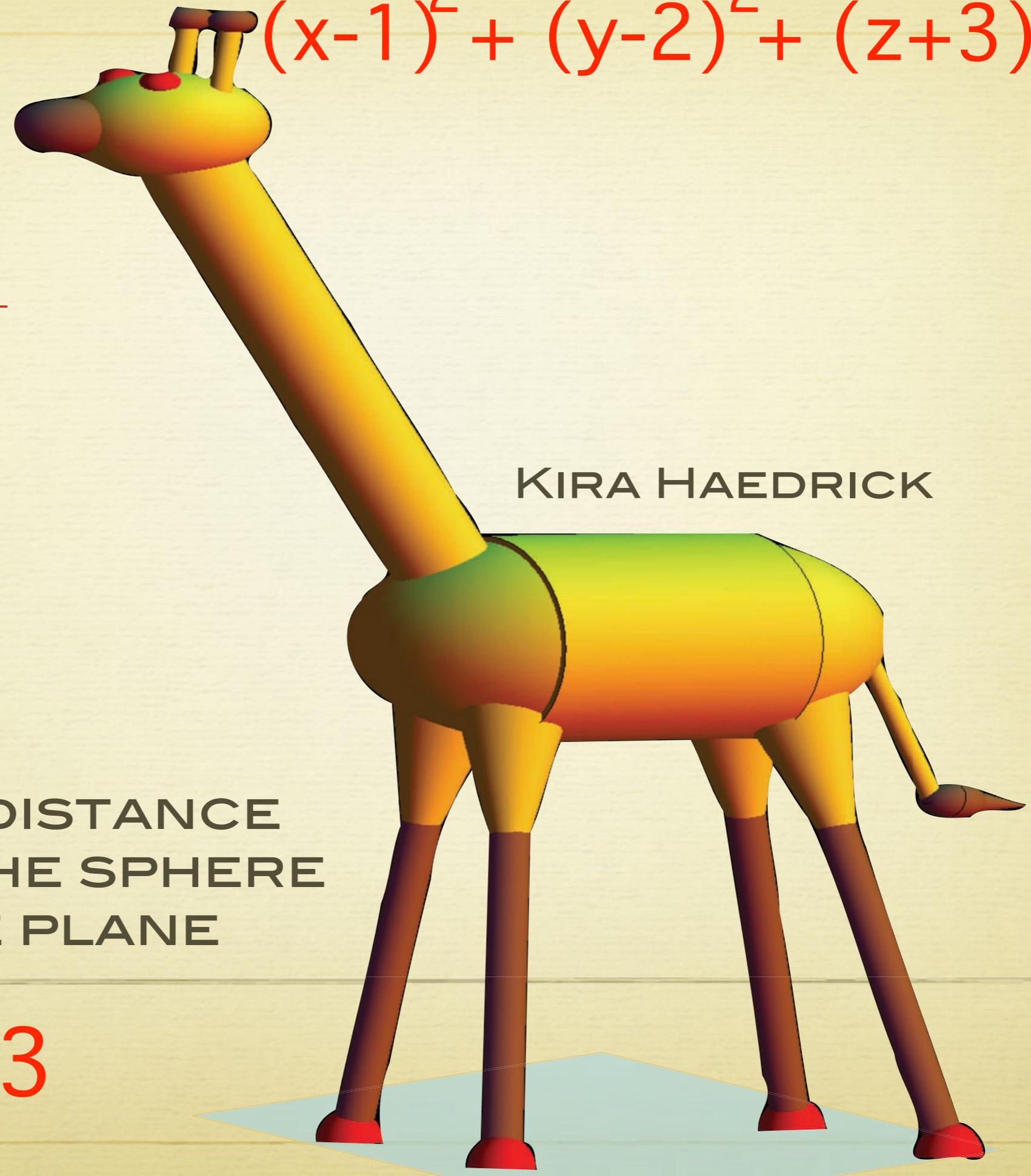
$$(x-1)^2 + (y-2)^2 + (z+3)^2 = 1$$

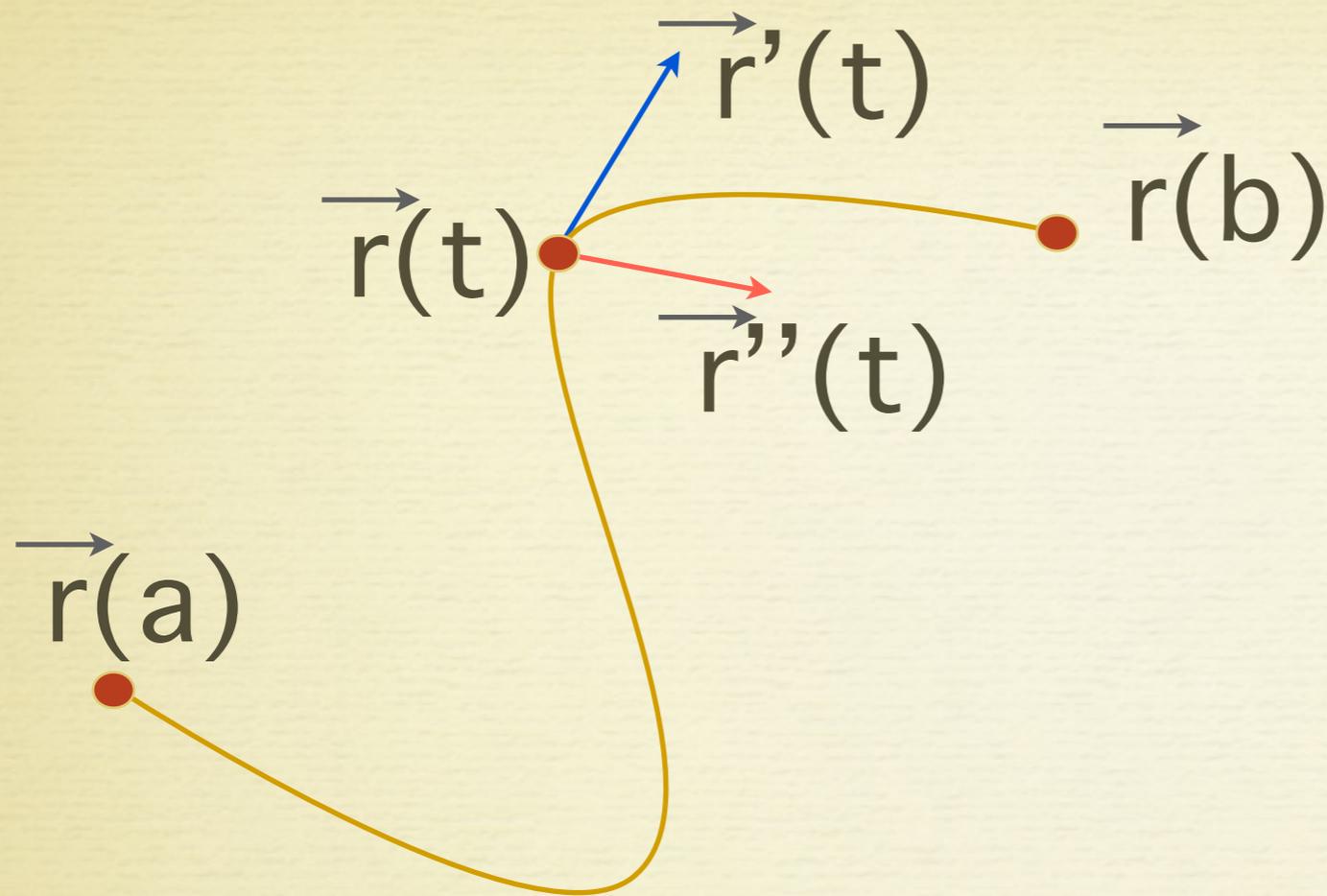
Problem

KIRA HAEDRICK

FIND THE DISTANCE
BETWEEN THE SPHERE
AND THE PLANE

$$x - y + z = 3$$





$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

$$\kappa(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}$$

Velocity and Acceleration

$$L = \int_a^b |\vec{r}'(t)| dt$$

Arc length

TNB

Concepts on Curves

curvature

$$\vec{r}(t) = \vec{r}(0) + t\vec{r}'(0) + \vec{R}(t)$$

$$\vec{v}(t) = \int_0^t \vec{r}''(s) ds, \vec{R}(t) = \int_0^t \vec{v}(s) ds$$

Integration

$$\nabla f(x,y,z) = \langle f_x, f_y, f_z \rangle$$

Gradient

$$D_{\vec{v}} f(x,y,z) = \langle f_x, f_y, f_z \rangle \cdot \vec{v}$$

Directional Derivative

$$\frac{d}{dt} f(\vec{r}(t)) = \nabla f(x,y,z) \cdot \vec{r}'(t)$$

Chain Rule

$|\nabla f|^2$ = directional derivative in the direction of the gradient

Steepest Ascent

$$\nabla f(x,y,z) = \langle a,b,c \rangle$$

normal vector

$$ax + by + cz = d$$

tangent plane

$$L(x,y,z) = f(x_0, y_0, z_0) = \\ a(x-x_0) + b(y-y_0) + c(z-z_0)$$

linearization

Estimate $f(x,y,z)$ by

$$L(x,y,z) \text{ near } (x_0, y_0, z_0)$$

estimation

these concepts are related

$$u_t(t,x) = u_x(t,x)$$

transport

$$u_t(t,x) = u_{xx}(t,x)$$

heat

$$u_{tt}(t,x) = u_{xx}(t,x)$$

wave

$$u_t + u u_x = u_{xx}$$

Burgers

$$u_{tx}(t,x) = u_{xt}(t,x)$$

Clairot

PDE's to know

HENRY COUSINS



© 2013 Cnes/Spot Image
Image Landsat
Image © 2013 DigitalGlobe

$$\nabla f(x,y) = \langle 0, 0 \rangle$$

$$D = f_{xx} f_{yy} - f_{xy}^2$$

$D > 0$	$f_{xx} < 0$	maximum
$D > 0$	$f_{xx} > 0$	minimum
$D < 0$		saddle point

Second Derivative Test

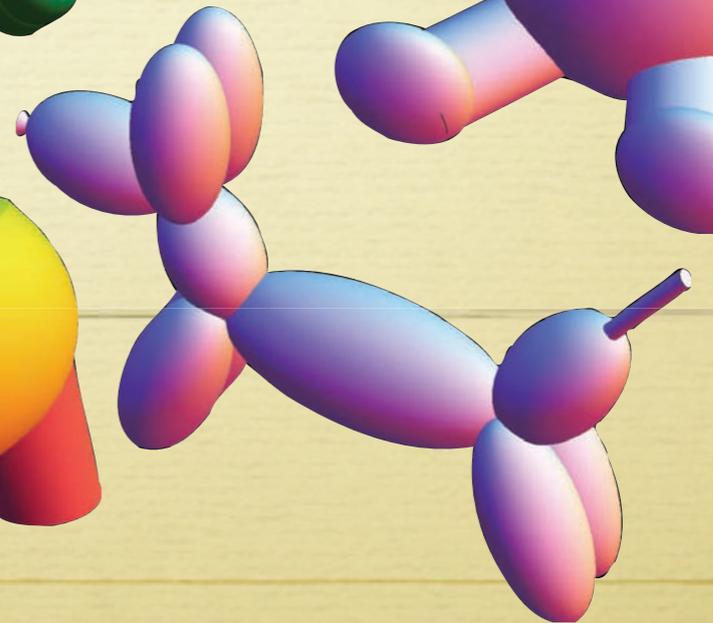
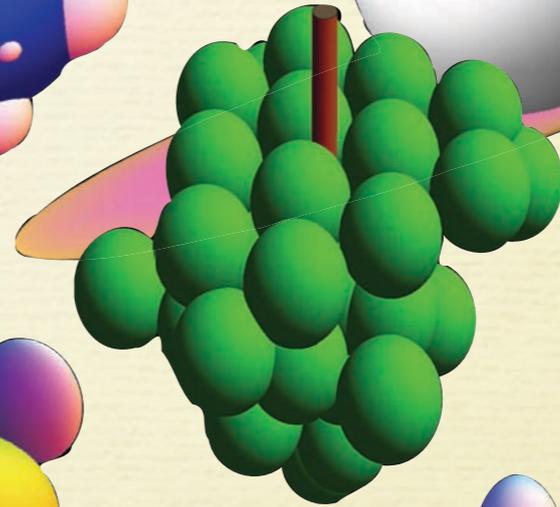
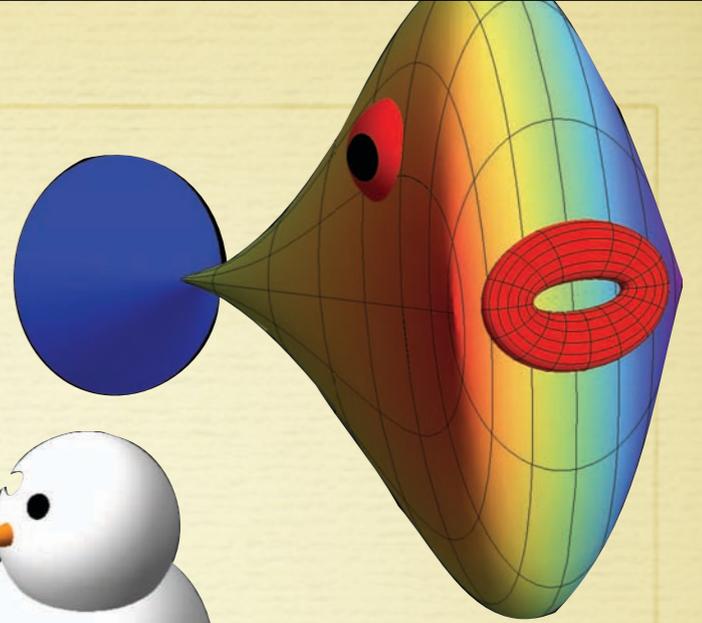
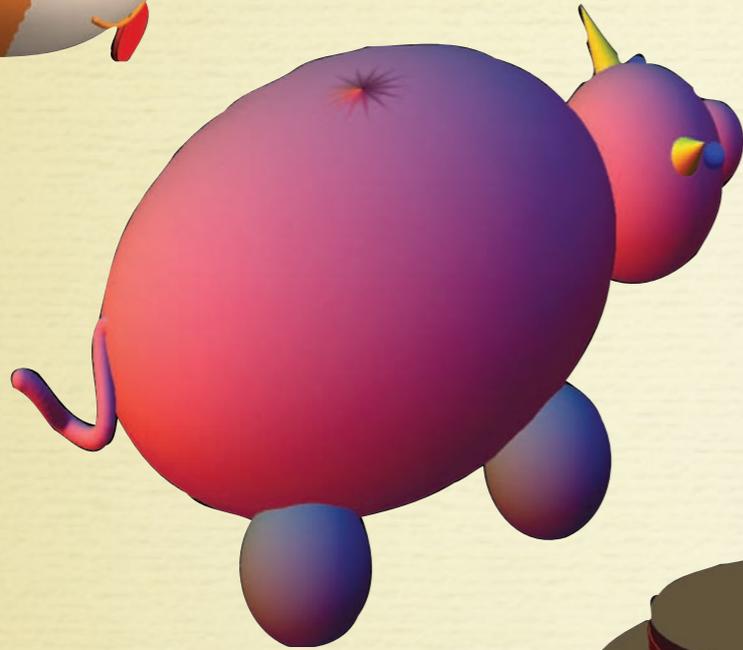
Problem

Where is the divergence of

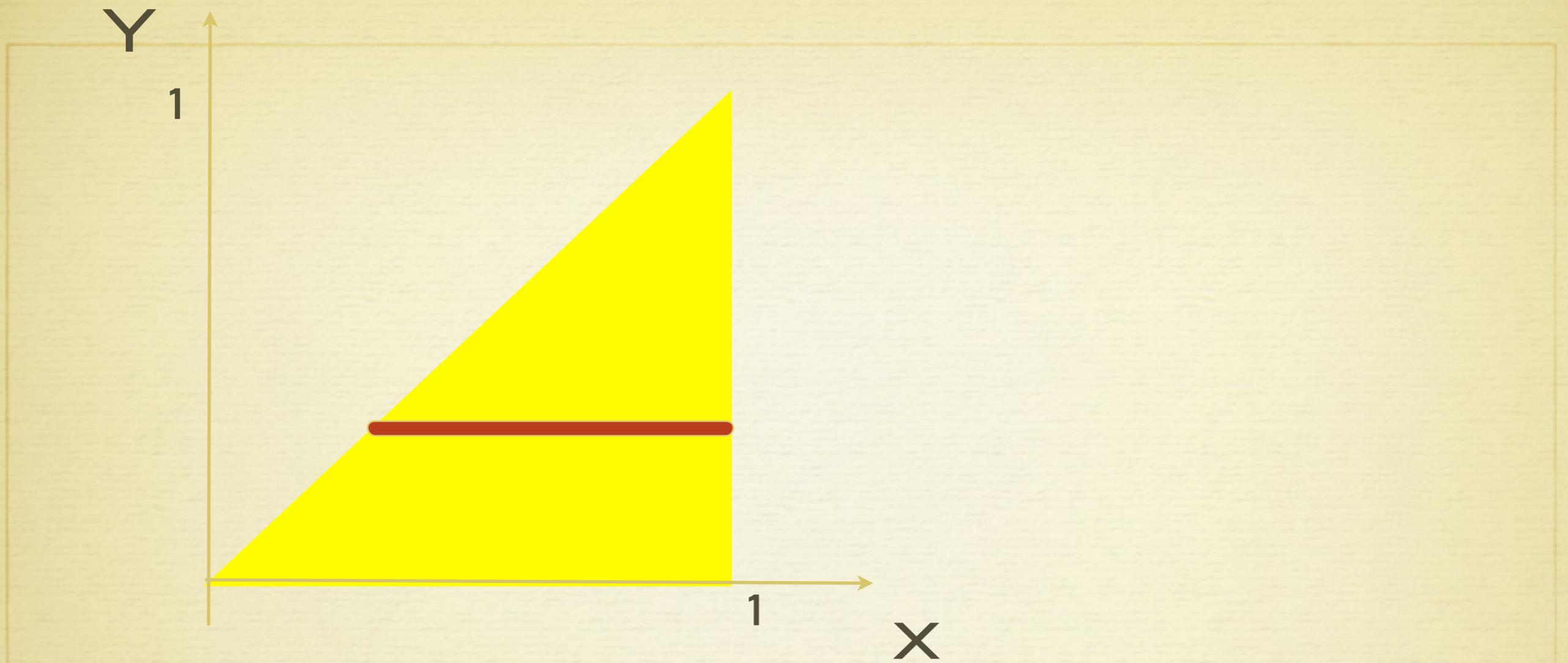
$$\vec{F}(x,y) = \langle x^3 y/3, 3 y^4/4 - 3 x y^2 \rangle$$

maximal and
where is it
minimal?

Marzipan to Win

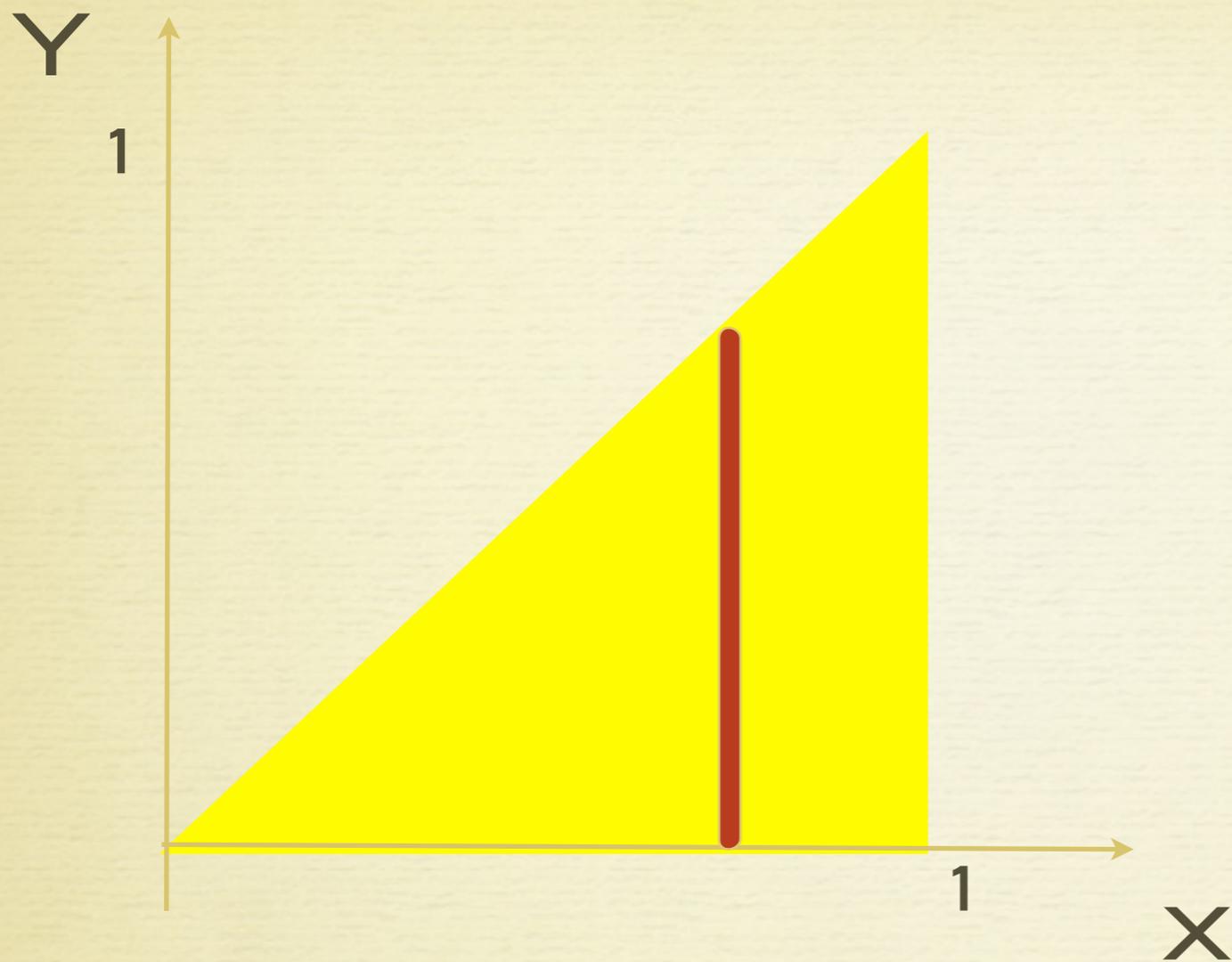


Shout the answer!



Example

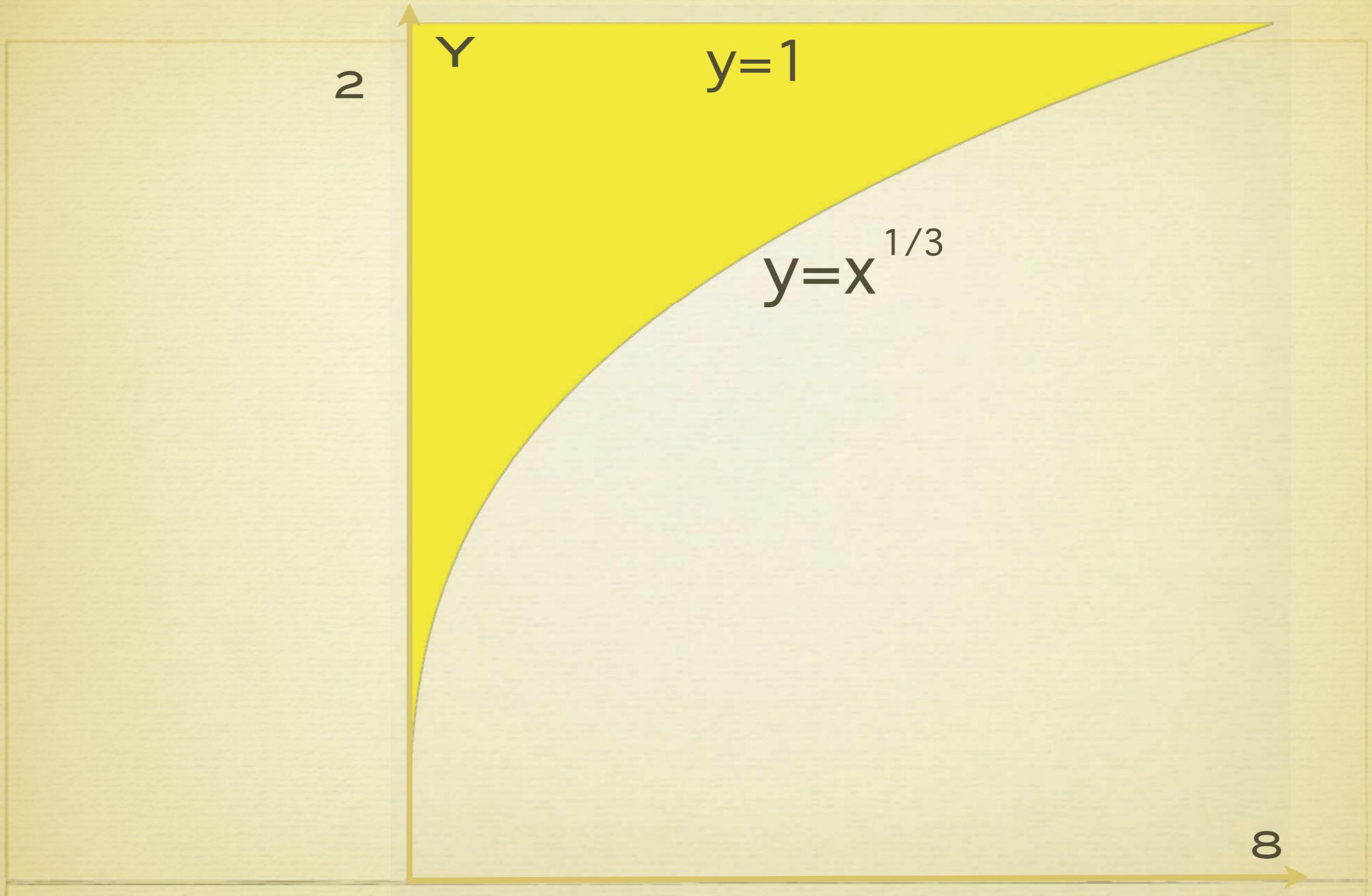
$$\int_0^1 \int_y^1 e^{-x^2} dx dy$$



$$\int_0^1 \int_0^x e^{-x^2} dy dx$$

$$\int_0^8 \int_{x^{1/3}}^2 \cos(y^4 + 1) \, dy \, dx$$

Problem



Make a picture

$$\int \int_R |\vec{r}_u \times \vec{r}_v| \, du \, dv$$

COMPARE:

$$\int_I |\vec{r}'(t)| \, dt$$

Surface area is an application of
double integrals Both are independent of parametrization!

What are the top 21a Mistakes?

