MATH 21A SECOND MIDTERM REVIEW

Selection of slides

Maths 21a, November 3, 2013

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PDE'S

An equation for an unknown function which involves derivatives with respect to at least two variables

$u_t = u_x$ Transport

$u_t = u_{xx}$ Heat

$u_{tt} = u_{xx}$ Wave

$u_t + u\, u_x = u_{xx}$ Burgers

$u_{xy} = u_{yx}$ Clairot

$u_{xx} + u_{yy} = 0$

Laplace

PROBLEM

$D_u f(x,y) = {}^{-}12$ $\quad u=(1,0)$

$D_v f(x,y) = {}^{-}4$ $\quad v=(3,4)/5$

$f(1,1) = 3$

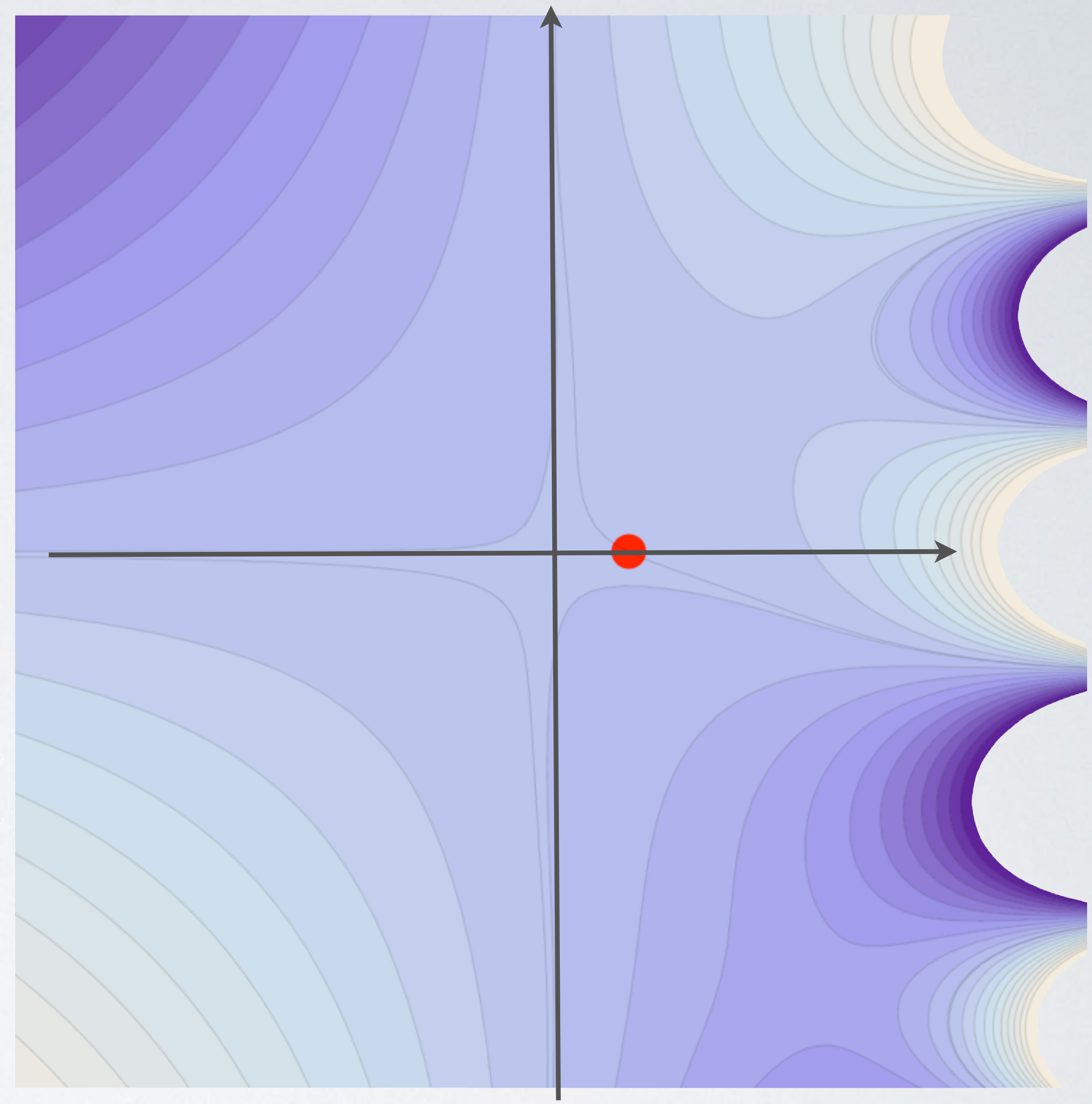
Can you estimate
$f(0.999,0.9999)$

PROBLEM

Find the tangent line to

$$2x\,y + e^{x-1}\cos(y) = 1$$

at (1,0)

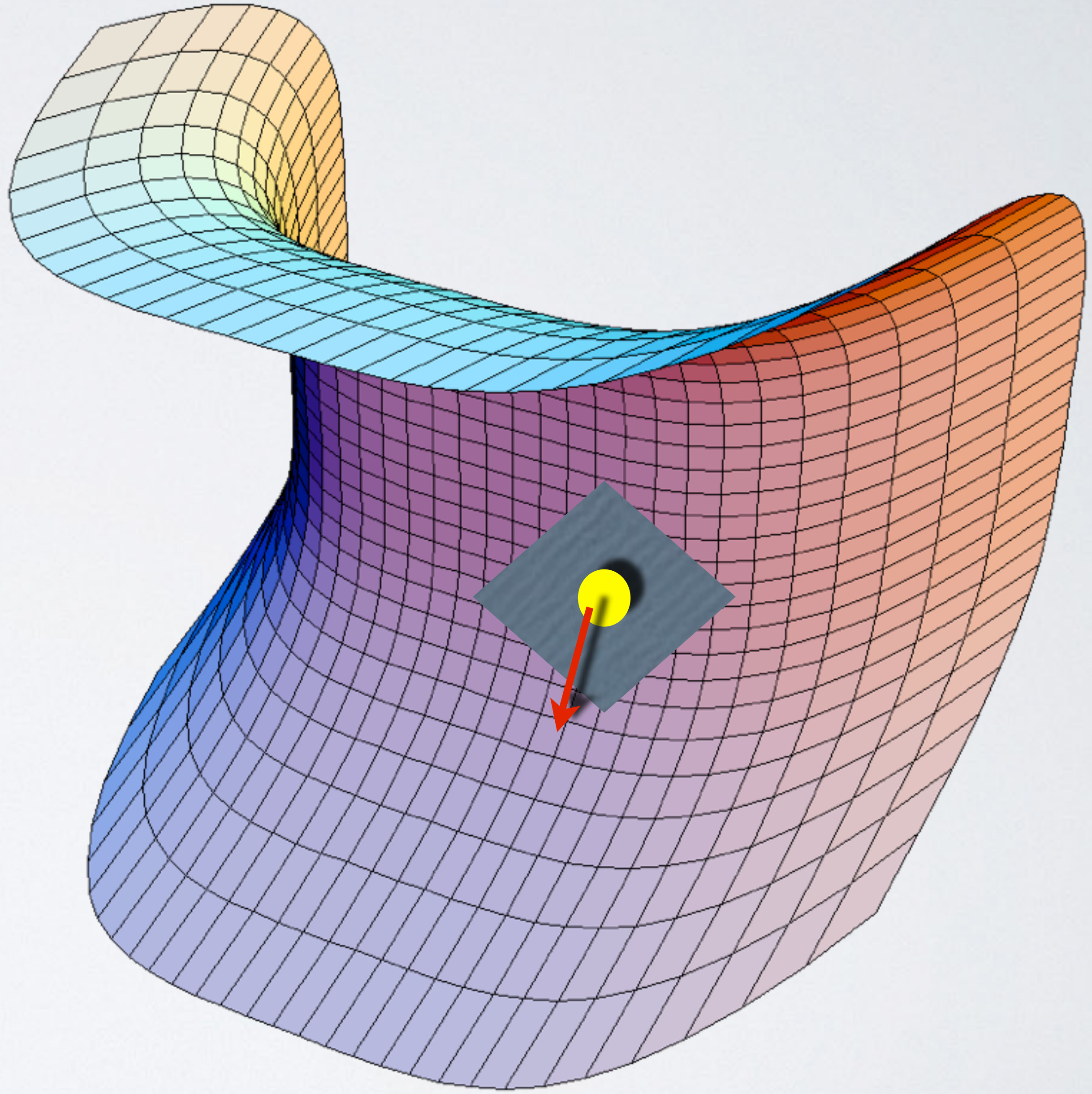


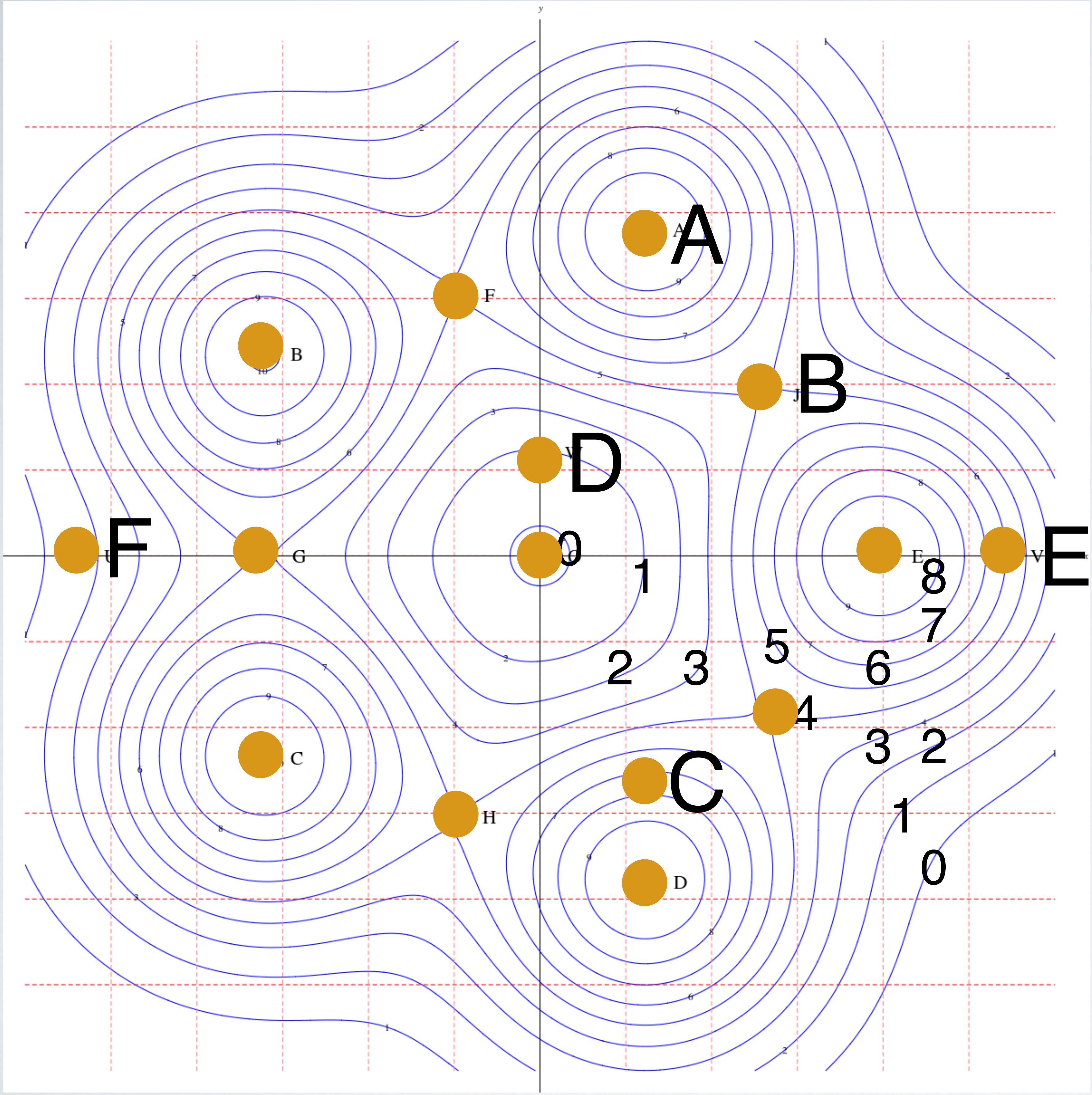
Problem

Find the **tangent plane** to the surface

$f(x,y,z) = x^4 - 2z^4 - y = 0$

at the point (1,0,1)





$f_x > 0, f_y = 0$

$f_x < 0, f_y = 0$

$f_x = 0, f_y > 0$

$f_x = 0, f_y < 0$

Saddle

Max

PROBLEM

We fly along

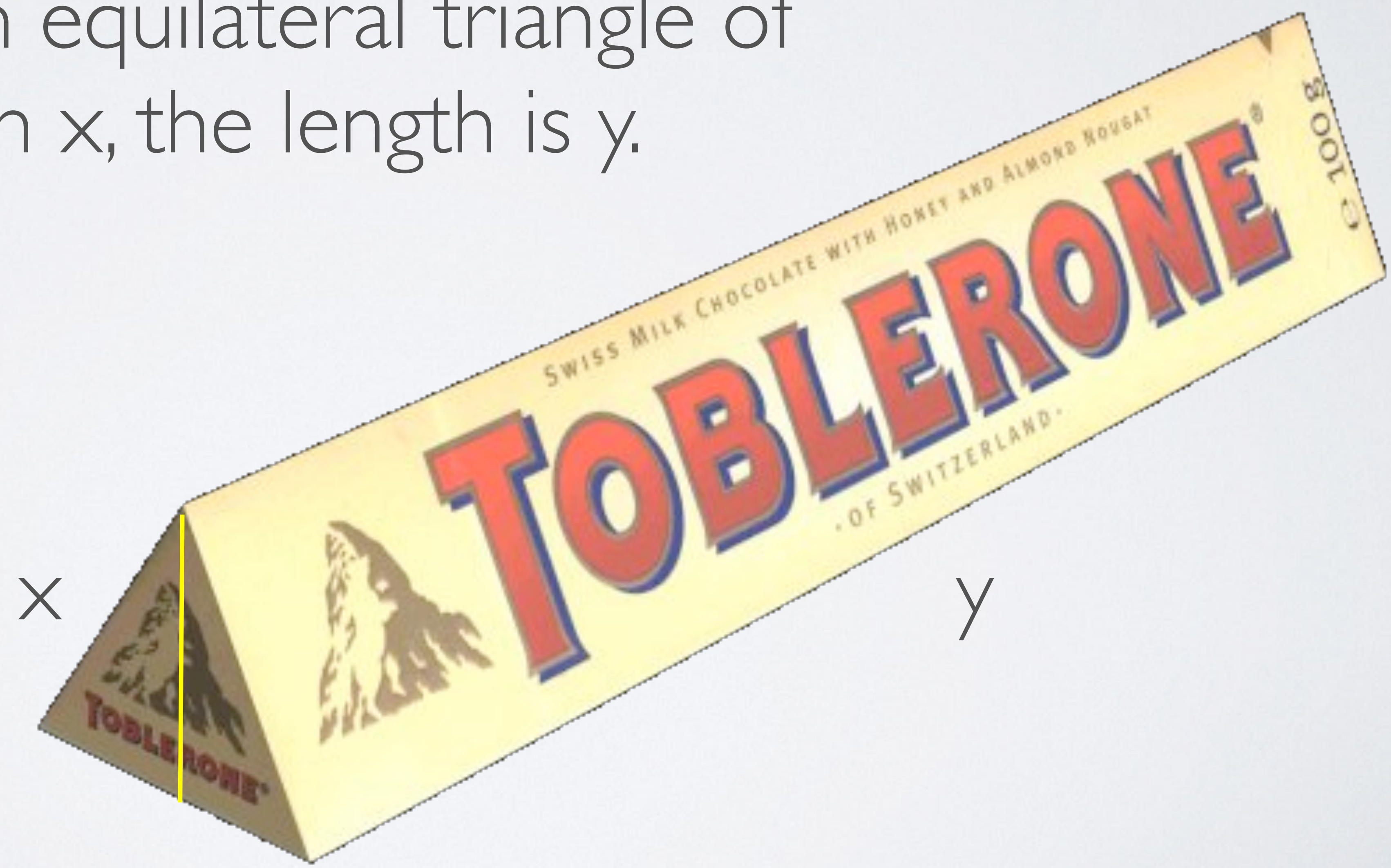
$r(t) = < t, 3\sin(t), 1-\cos(t)>$.

How fast does the distance to the origin change at t=0?

TOBLERONE PROBLEM

Find the toblerone chocolate shape which has maximal volume if the surface area is constant 1.

The base is an equilateral triangle of side length x, the length is y.



$$g = 3xy + x^2\frac{\sqrt{3}}{2}$$

$$f = x^2 y \frac{\sqrt{3}}{4}$$

1 / 2

plug into 3

1

2

3

x

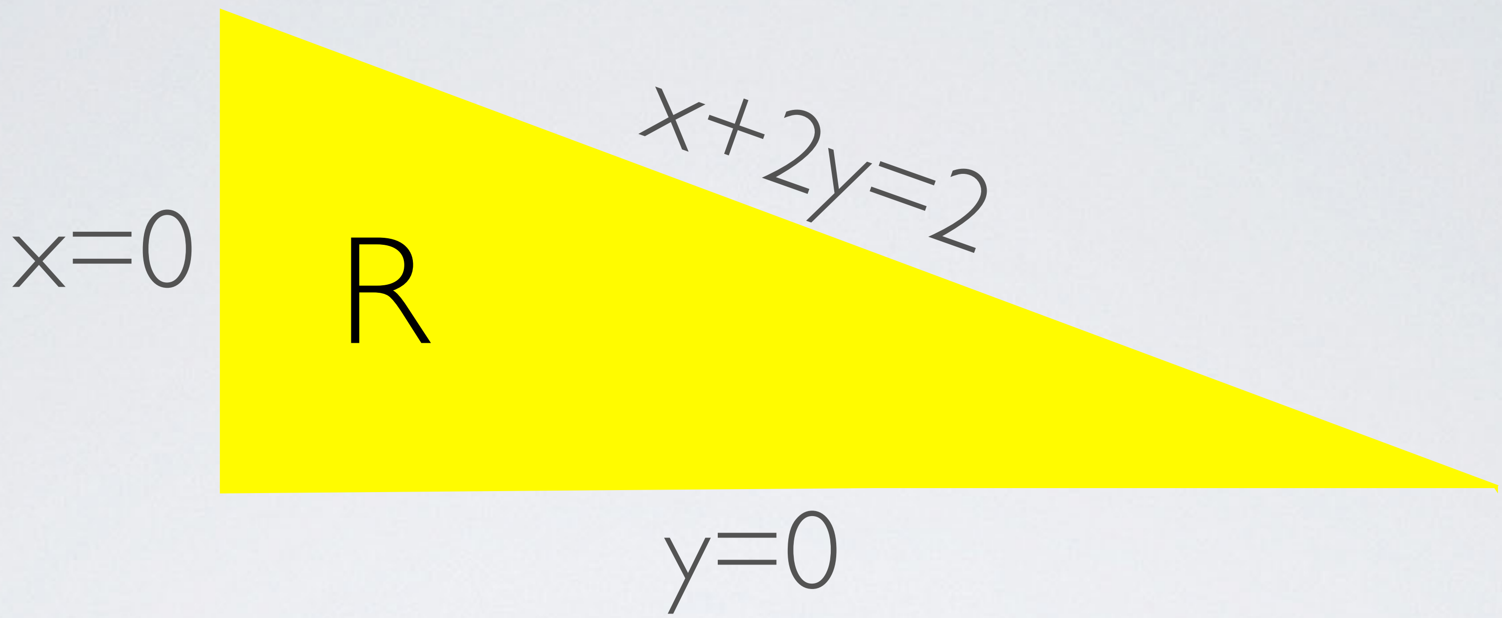
y

IS THERE GLOBAL MAX?

$$f(x,y) = y^2 x^2 + y^4$$

PROBLEM:

Integrate $\exp(y)/(y-1)$ over the region bound by $x+2y=2$, $y=0$ and $x=0$!

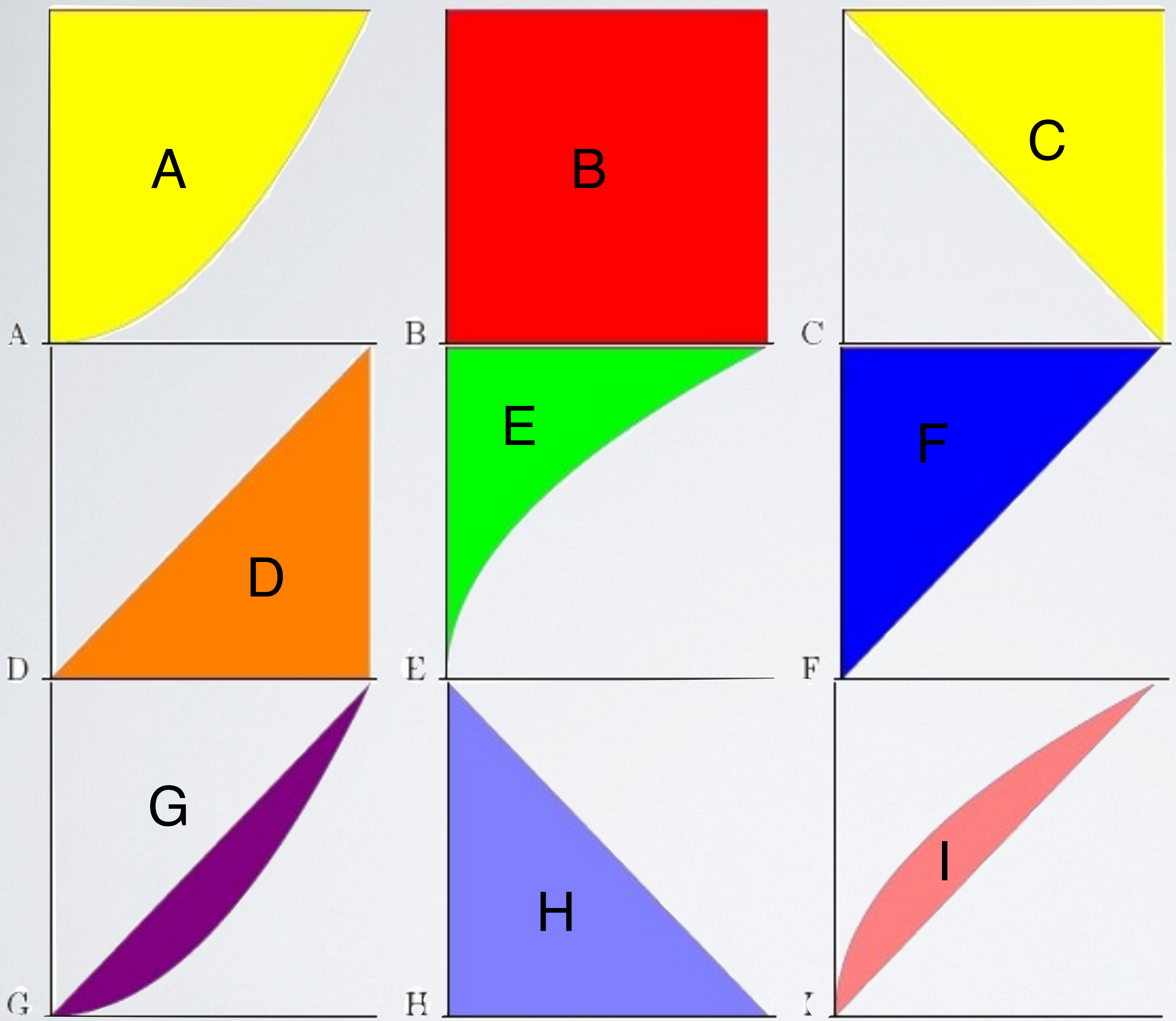


As a type I region

$$\int_0^2 \int_0^{x/2-1} e^y /(y-1)\, dy\, dx$$

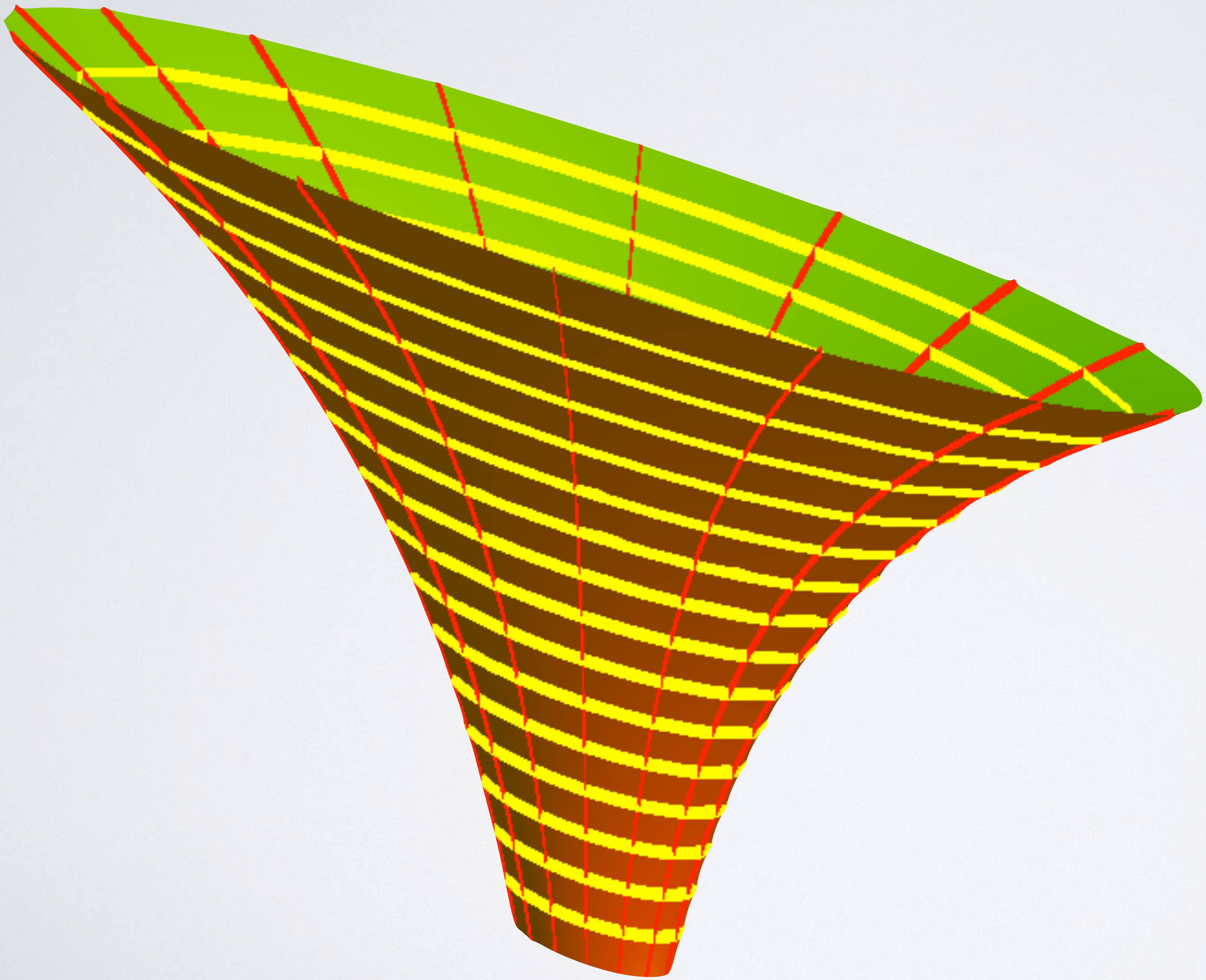
As a type II region

$$\int_0^1 \int_0^{2-2y} e^y /(y-1)\, dx\, dy$$



A-I	Integral
	$\int_0^2 \int_{x^2/2}^2 f(x,y)\, dydx$
	$\int_0^2 \int_0^x f(x,y)\, dydx$
	$\int_0^2 \int_0^2 f(x,y)\, dydx$
	$\int_0^2 \int_0^{y^2/2} f(x,y)\, dxdy$
	$\int_0^2 \int_0^y f(x,y)\, dxdy$
	$\int_0^2 \int_{y^2/2}^y f(x,y)\, dxdy$
	$\int_0^2 \int_{x^2/2}^x f(x,y)\, dydx$
	$\int_0^2 \int_0^{2-x} f(x,y)\, dydx$
	$\int_0^2 \int_{2-x}^2 f(x,y)\, dydx$

FIND THE SURFACE AREA



$$x^2 + y^2 = e^{2z}$$

$$0 < z < 1$$