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MWF 12 Steve Wang
MWF 14 Mike Hopkins
TTH 10 Oliver Knill
TTH 10 Francesco Cavazzani
TTH 11:30 Kate Penner
TTH 11:30 Francesco Cavazzani

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

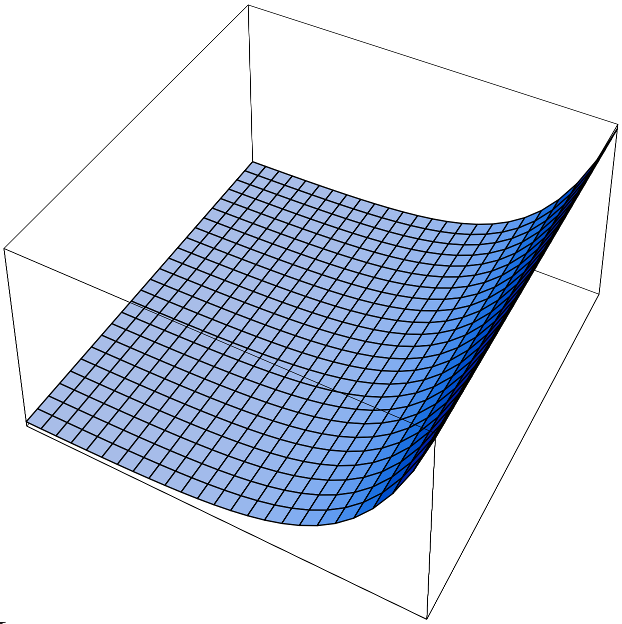
Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

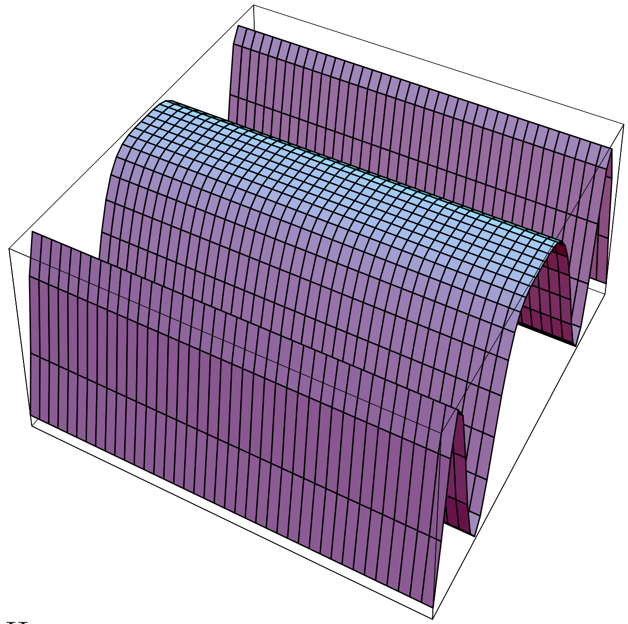
1)	T	F	The point $(x, y, z) = (-1, -1, -1)$ is in spherical coordinate described as $(\rho, \theta, \phi) = (\sqrt{3}, 5\pi, 3\pi/4)$
2)	T	F	If $\lvert\vec{v} \times \vec{w}\rvert = 0$ then $\vec{v} = \vec{0}$ or $\vec{w} = \vec{0}$.
3)	T	F	The surface $z^2 + 4y^2 = x^2 + 1$ is a two sheeted hyperboloid.
4)	T	F	The surface $4x^2 - 4x + y^2 - 2y - 120 = -z^2$ is an ellipsoid.
5)	T	F	The parametrized lines $\vec{u}(t) = \langle 1 + 2t, 2 - 5t, 1 + t \rangle$ and $\vec{v}(t) = \langle 3 - 4t, -3 + 10t, 2 - 2t \rangle$ are the same line.
6)	T	F	The surface $\sin(x) = z$ contains lines which are parallel to the y-axis.
7)	T	F	If $\vec{u} \cdot \vec{v} = 0$, $\vec{v} \cdot \vec{w} = 0$ and $\vec{v}$ is not the zero vector, then $\vec{u} \cdot \vec{w} = 0$.
8)	T	F	The curvature of a curve depends upon the speed at which one travels upon it.
9)	T	F	Two lines in space that do not intersect must be parallel.
10)	T	F	A line in space can intersect an elliptic paraboloid in 4 points.
11)	T	F	If $\vec{u} \times \vec{v} = 0$ and $\vec{u} \cdot \vec{v} = 0$, then one of the vectors $\vec{u}$ and $\vec{v}$ is zero.
12)	T	F	If the velocity vector $\vec{r}\,'(t)$ and the acceleration vector $\vec{r}\,''(t)$ of a curve are parallel at time $t = 1$, then the curvature $\kappa(t)$ of the curve is zero at time $t = 1$.
13)	T	F	If the speed of a parametrized curve is constant over time, then the curvature of the curve $\vec{r}(t)$ is zero.
14)	T	F	The length of the vector projection of a vector $\vec{v}$ onto a vector $\vec{w}$ is always equal to the length of the vector projection of $\vec{w}$ onto $\vec{v}$.
15)	T	F	A quadric $ax^2 + by^2 + cz^2 = 1$ is contained in the interior of a sphere $x^2 + y^2 + z^2 < 100$, then the constants a, b, c are all positive and the quadric is an ellipsoid.
16)	T	F	There is a hyperboloid of the form $ax^2 + by^2 - cz^2 = 1$ which has a trace which is a parabola.
17)	T	F	The set of points in space which have distance 1 from the line $x = y = z$ form a cylinder.
18)	T	F	The velocity vector of a parametric curve $\vec{r}(t)$ always has constant length.
19)	T	F	The volume of a parallelepiped spanned by $\vec{u}, \vec{v}, \vec{w}$ is $\lvert(\vec{u} \times \vec{v}) \times \vec{w}\rvert$.
20)	T	F	The equation $x^2 + y^2/4 = 1$ in space describes an ellipsoid.

Problem 2a) (3 points)

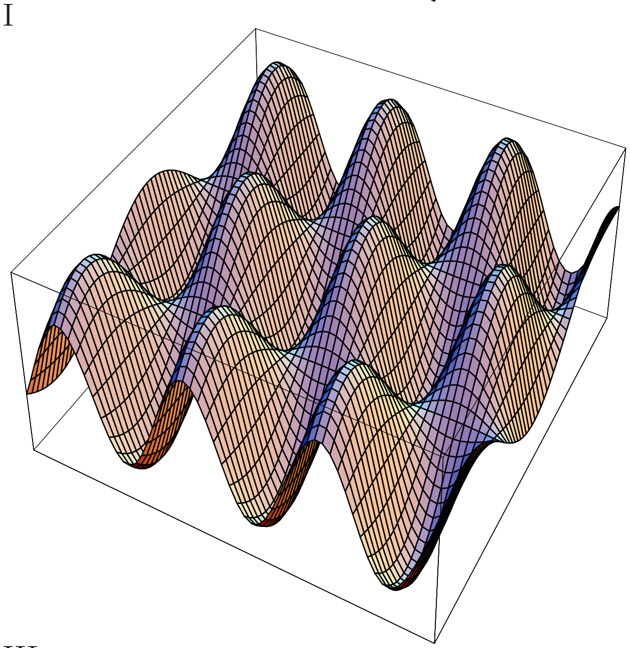
Match the equation with their graphs. No justifications are needed.



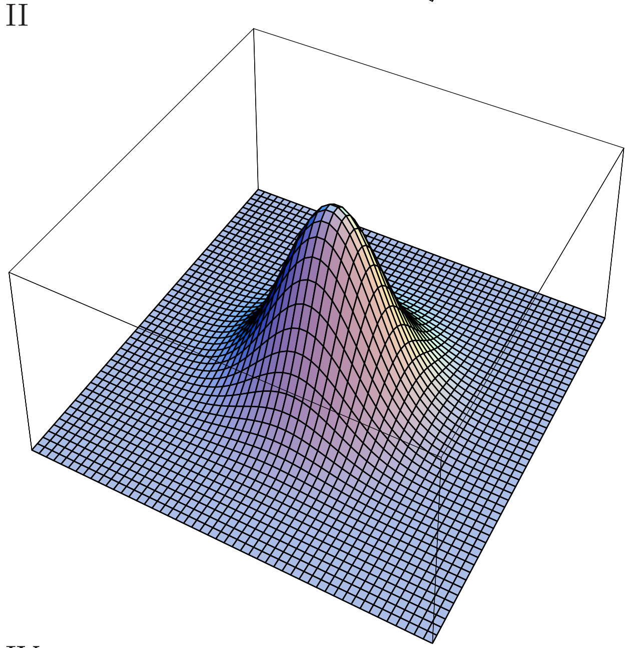
I



II



III



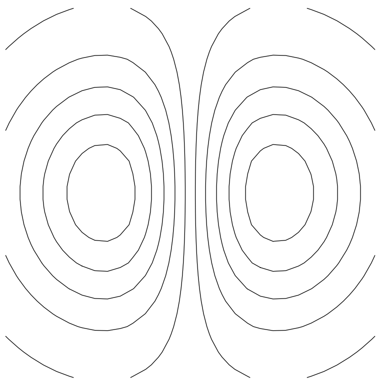
IV

Enter I,II,III,IV here	Equation
	$z = \sin(5x)\cos(2y)$
	$z = \cos(y^2)$
	$z = e^{-x^2-y^2}$
	$z = e^x$

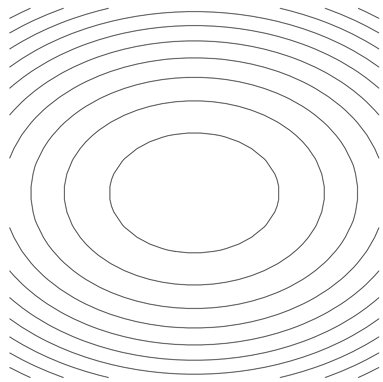
Problem 2b) (4 points)

Match the contour maps with the corresponding functions $f(x,y)$ of two variables. No justifications are needed.

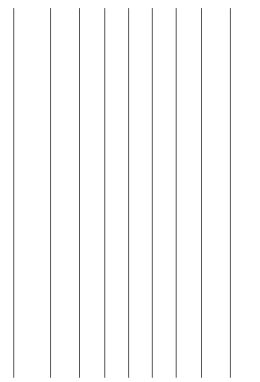
I



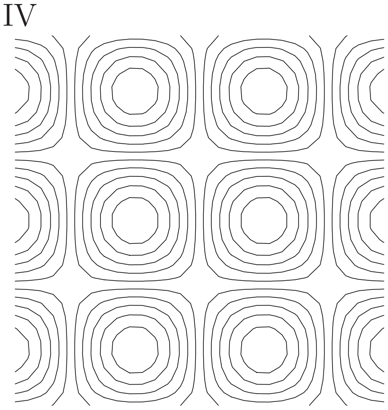
II



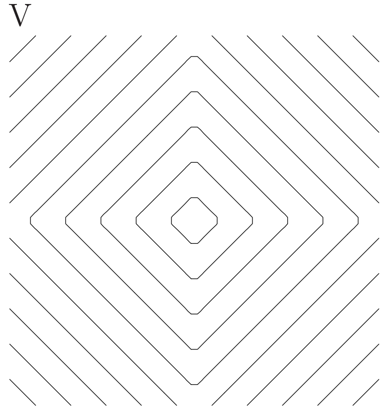
III



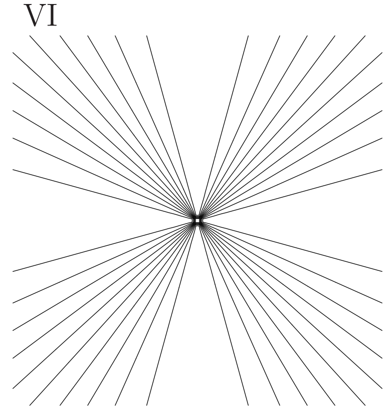
IV



V



VI



Enter I,II,III,IV,V or VI here	Function $f(x,y)$
	$f(x,y) = \sin(x)$
	$f(x,y) = x^2 + 2y^2$
	$f(x,y) = \|x\| + \|y\|$
	$f(x,y) = \sin(x)\cos(y)$
	$f(x,y) = xe^{-x^2-y^2}$
	$f(x,y) = x^2/(x^2+y^2)$

Problem 2c) (3 points)

Match the following points in cartesian coordinates with the points in spherical coordinates:

a) $(x, y, z) = (\sqrt{2}, 0, 0)$
b) $(x, y, z) = (0, \sqrt{2}, 0)$
c) $(x, y, z) = (0, 0, \sqrt{2})$
d) $(x, y, z) = (1, 1, 0)$
e) $(x, y, z) = (1, 0, 1)$
f) $(x, y, z) = (0, 1, 1)$

1) $(\rho, \phi, \theta) = (\sqrt{2}, 0, 0)$.
2) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, \pi/4)$.
3) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, 0)$.
4) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/2, \pi/2)$.
5) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/4, \pi/2)$.
6) $(\rho, \phi, \theta) = (\sqrt{2}, \pi/4, 0)$.

Problem 3) (10 points)

a) (7 points) Find a parametric equation for the line which is the intersection of the two planes $2x - y + 3z = 9$ and $x + 2y + 3z = -7$.

b) (3 points) Find a plane perpendicular to both planes given in a) which has the additional property that it passes through the point $P = (1, 1, 1)$.

Problem 4) (10 points)

Given the vectors $\vec{v} = \langle 1, 1, 0 \rangle$ and $\vec{w} = \langle 0, 0, 1 \rangle$ and the point $P = (2, 4, -2)$. Let Σ be the plane which goes through the origin $(0, 0, 0)$ and which contains the vectors $\vec{v}$ and $\vec{w}$. Let S be the unit sphere $x^2 + y^2 + z^2 = 1$.

a) (6 points) Compute the distance from P to the plane Σ.

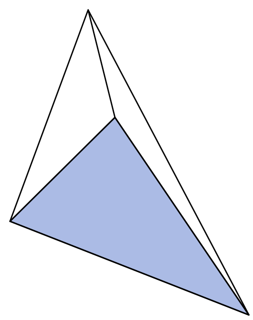
b) (4 points) Find the shortest distance from P to the sphere S.

Problem 5) (10 points)

a) (6 points) Find an equation for the plane through the points $A = (0, 1, 0), B = (1, 2, 1)$ and $C = (2, 4, 5)$.

b) (4 points) Given an additional point $P = (-1, 2, 3)$, what is the volume of the tetrahedron which has A, B, C, P among its vertices.

A useful fact which you can use without justification in b): the volume of the tetrahedron is 1/6 of the volume of the parallelepiped which has AB, AC, and AP among its edges.



Problem 6) (10 points)

The parametrized curve $\vec{u}(t) =< t, t^2, t^3 >$ (known as the "twisted cubic") intersects the parametrized line $\vec{v}(s) = \langle 1 + 3s, 1 - s, 1 + 2s \rangle$ at a point P. Find the angle of intersection.

Problem 7) (10 points)

Let $\vec{r}(t)$ be the space curve $\vec{r}(t) = (\log(t), 2t, t^2)$, where $\log(t)$ is the natural logarithm (denoted by $\ln(t)$ in some textbooks).

a) What is the velocity and what is the acceleration at time $t = 1$?

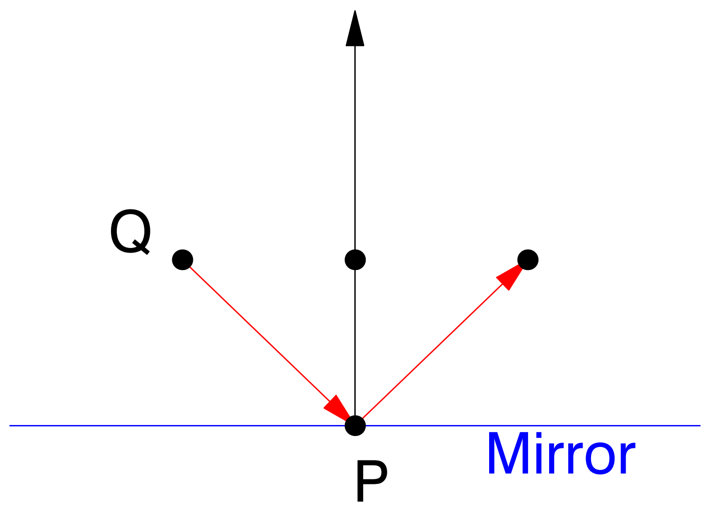
b) Find the length of the curve from $t = 1$ to $t = 2$.

Problem 8) (10 points)

A planar mirror in space contains the point $P = (4, 1, 5)$ and is perpendicular to the vector $\vec{n} = \langle 1, 2, -3 \rangle$. The light ray $\vec{QP} = \vec{v} = \langle -3, 1, -2 \rangle$ with source $Q = (7, 0, 7)$ hits the mirror plane at the point P.

a) (4 points) Compute the projection $\vec{u} = \vec{\mathrm{P}}_{\vec{n}}(\vec{v})$ of $\vec{v}$ onto $\vec{n}$.

b) (6 points) Identify $\vec{u}$ in the figure and use it to find a vector parallel to the reflected ray.



Problem 9) (10 points)

We know the acceleration $\vec{r}\,''(t) = \langle 2, 1, 3 \rangle + t\langle 1, -1, 1 \rangle$ and the initial position $\vec{r}(0) = \langle 0, 0, 0 \rangle$ and initial velocity $\vec{r}\,'(0) = \langle 11, 7, 0 \rangle$ of an unknown curve $\vec{r}(t)$. Find $\vec{r}(6)$.

Problem 10) (10 points)

Intersecting the elliptic cylinder $x^2 + y^2/4 = 1$ with the plane $z = \sqrt{3}x$ gives a curve in space.

a) (3 points) Find the parametrization of the curve.

b) (3 points) Compute the unit tangent vector $\vec{T}$ to the curve at the point $(0, 2, 0)$.

c) (4 points) Write down the arc length integral and evaluate the arc length of the curve.