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TTH 11:30 Kate Penner

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- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work.** Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

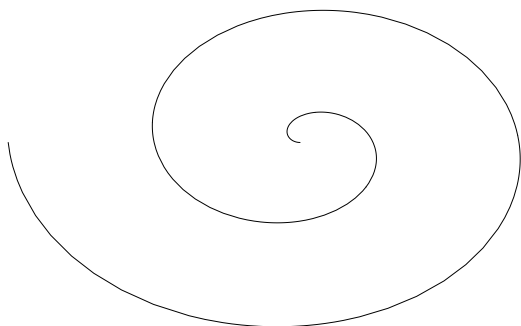
Problem 1) TF questions (20 points) No justifications needed

- | | | |
|-----|---|--|
| 1) | ☐ T ☐ F | The length of the sum of two vectors is always the sum of the length of the vectors. |
| 2) | ☐ T ☐ F | For any three vectors, $\vec{v} \times (\vec{w} + \vec{u}) = \vec{w} \times \vec{v} + \vec{u} \times \vec{v}$. |
| 3) | ☐ T ☐ F | The set of points which satisfy $x^2 + 2x + y^2 - z^2 = 0$ is a cone. |
| 4) | ☐ T ☐ F | The surface $\vec{r}(u, v) = \langle \cos(u^2) \sin(v^2), \sin(u^2) \sin(v^2), \cos(v^2) \rangle$ with $0 \leq u < \sqrt{2\pi}, 0 \leq v \leq \sqrt{\pi}$ is a sphere. |
| 5) | ☐ T ☐ F | If P, Q, R are 3 different points in space that don't lie in a line, then $\vec{PQ} \times \vec{RQ}$ is a vector orthogonal to the plane containing P, Q, R . |
| 6) | ☐ T ☐ F | The line $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$ hits the plane $2x + 3y + 4z = 9$ at a right angle. |
| 7) | ☐ T ☐ F | The function $f(x, y) = \sin(xy)/y$ is continuous everywhere. |
| 8) | ☐ T ☐ F | For any two vectors, $\vec{v} \times \vec{w} = \vec{w} \times \vec{v}$. |
| 9) | ☐ T ☐ F | If $ \vec{v} \times \vec{w} = 0$ for all vectors $\vec{w}$, then $\vec{v} = \vec{0}$. |
| 10) | ☐ T ☐ F | If $\vec{u}$ and $\vec{v}$ are orthogonal vectors, then $(\vec{u} \times \vec{v}) \times \vec{u}$ is parallel to $\vec{v}$. |
| 11) | ☐ T ☐ F | Every vector contained in the plane $x + y + z = 1$ is parallel to the vector $\langle 1, 1, 1 \rangle$. |
| 12) | ☐ T ☐ F | The sphere can in cylindrical coordinates described as $r^2 = 1 - z^2$. |
| 13) | ☐ T ☐ F | The curvature of the curve $2\vec{r}(4t)$ at $t = 0$ is twice the curvature of the curve $\vec{r}(t)$ at $t = 0$. |
| 14) | ☐ T ☐ F | The set of points which satisfy $x^2 - 2y^2 - 3z^2 = 0$ form an ellipsoid. |
| 15) | ☐ T ☐ F | If $\vec{v} \times \vec{w} = (0, 0, 0)$, then $\vec{v} = \vec{w}$. |
| 16) | ☐ T ☐ F | Every vector contained in the line $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$ is parallel to the vector $\langle 1, 1, 1 \rangle$. |
| 17) | ☐ T ☐ F | Two nonzero vectors are parallel if and only if their cross product is $\vec{0}$. |
| 18) | ☐ T ☐ F | The vector $\vec{u} \times (\vec{v} \times \vec{w})$ is always in the same plane together with $\vec{v}$ and $\vec{w}$. |
| 19) | ☐ T ☐ F | The line $\vec{r}(t) = \langle 1 + 2t, 1 + 2t, 1 - 4t \rangle$ hits the plane $x + y + z = 9$ at a right angle. |
| 20) | ☐ T ☐ F | The intersection of the ellipsoid $x^2/3 + y^2/4 + z^2/3 = 1$ with the plane $y = 1$ is a circle. |

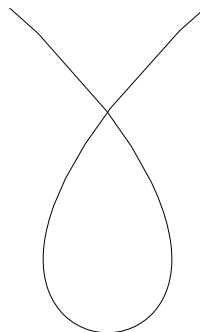
Problem 2a) (3 points)

Match the curves with their parametric definitions.

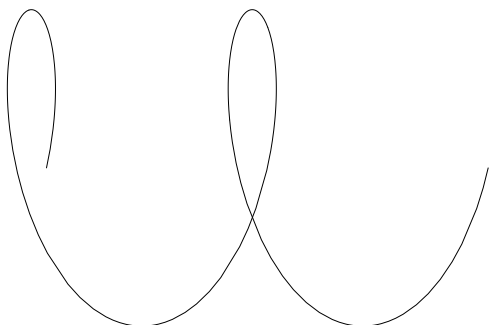
I



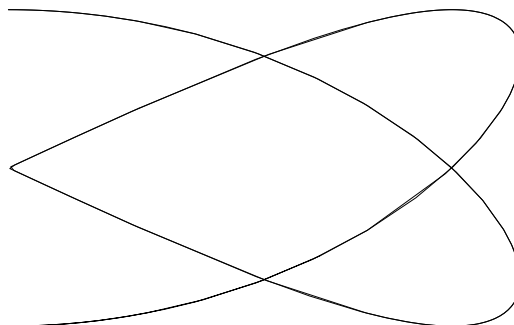
II



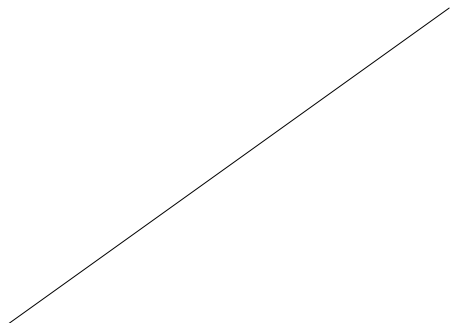
III



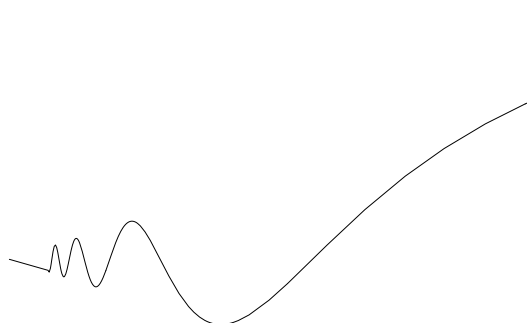
IV



V

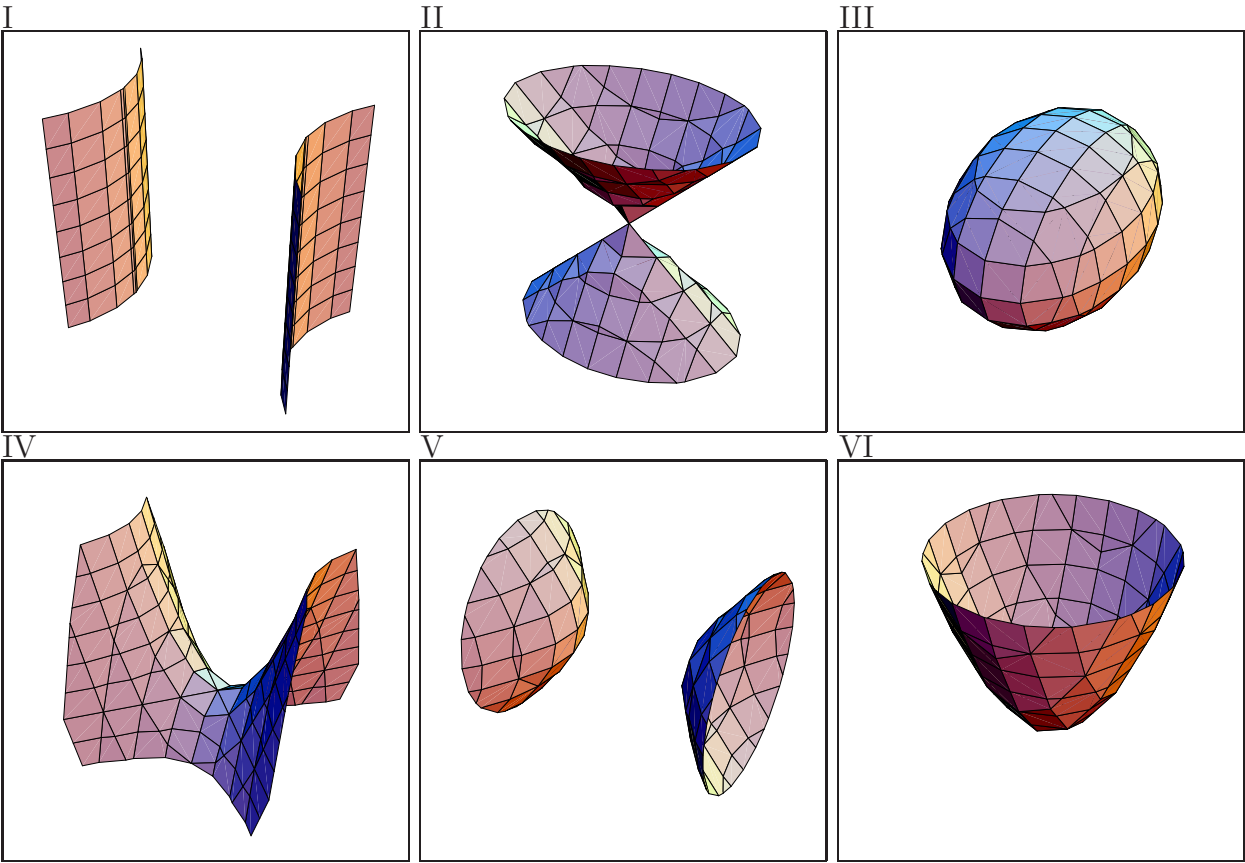


VI



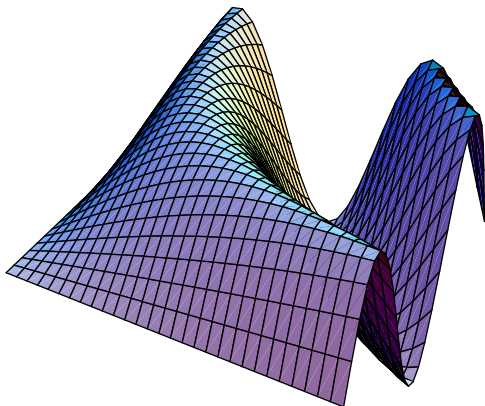
Enter I,II,III,IV,V or VI here	Parametric equation for the curve
	$\vec{r}(t) = \langle t, \sin(1/t)t \rangle$
	$\vec{r}(t) = \langle t^3 - t, t^2 \rangle$
	$\vec{r}(t) = \langle t + \cos(2t), \sin(2t) \rangle$
	$\vec{r}(t) = \langle \sin(2t) , \cos(3t) \rangle$
	$\vec{r}(t) = \langle 1 + t, 5 + 3t \rangle$
	$\vec{r}(t) = \langle -t \cos(t), 2t \sin(t) \rangle$

Match the equations with the surfaces.

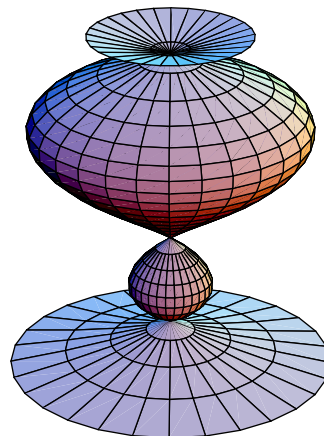


Enter I,II,III,IV,V,VI here	Equation
	$x^2 - y^2 - z^2 = 1$
	$x^2 + 2y^2 = z^2$
	$2x^2 + y^2 + 2z^2 = 1$
	$x^2 - y^2 = 5$
	$x^2 - y^2 - z = 1$
	$x^2 + y^2 - z = 1$

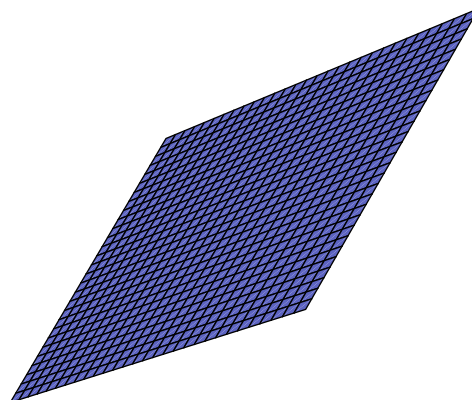
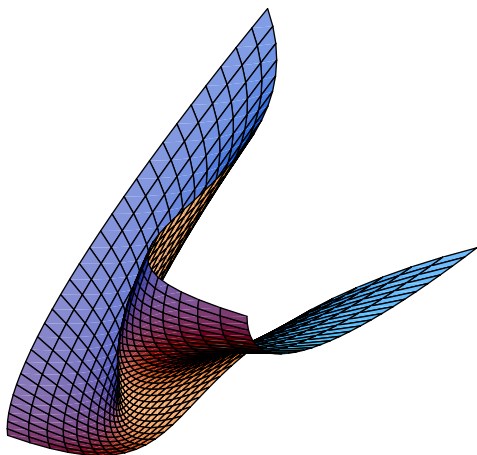
Match the parametric surfaces with their parameterization. No justification is needed.



I



II



III

Enter I,II,III,IV here	Parameterization
	$\vec{r}(u, v) = \langle u, v, u + v \rangle$
	$\vec{r}(u, v) = \langle u, v, \sin(uv) \rangle$
	$\vec{r}(u, v) = \langle 0.2 + u(1 - u^2) \cos(v), (0.2 + u(1 - u^2)) \sin(v), u \rangle$
	$\vec{r}(u, v) = \langle u^3, (u - v)^2, v \rangle$

Problem 3) (10 points)

a) (6 points) Find a parameterization of the line of intersection of the planes $3x - 2y + z = 7$ and $x + 2y + 3z = -3$.

b) (4 points) Find the symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

representing that line.

Problem 4) (10 points)

a) (4 points) Find the area of the parallelogram with vertices $P = (1, 0, 0)$ $Q = (0, 2, 0)$, $R = (0, 0, 3)$ and $S = (-1, 2, 3)$.

b) (3 points) Verify that the triple scalar product has the property $[\vec{u} + \vec{v}, \vec{v} + \vec{w}, \vec{w} + \vec{u}] = 2[\vec{u}, \vec{v}, \vec{w}]$.

c) (3 points) Verify that the triple scalar product $[\vec{u}, \vec{v}, \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$ has the property

$$|[\vec{u}, \vec{v}, \vec{w}]| \leq \|\vec{u}\| \cdot \|\vec{v}\| \cdot \|\vec{w}\|$$

Problem 5) (10 points)

Find the distance between the two lines

$$\vec{r}_1(t) = \langle t, 2t, -t \rangle$$

and

$$\vec{r}_2(t) = \langle 1 + t, t, t \rangle .$$

Problem 6) (10 points)

Find an equation for the plane that passes through the origin and whose normal vector is parallel to the line of intersection of the planes $2x + y + z = 4$ and $x + 3y + z = 2$.

Problem 7) (10 points)

The intersection of the two surfaces $x^2 + \frac{y^2}{2} = 1$ and $z^2 + \frac{y^2}{2} = 1$ consists of two curves.

- a) (4 points) Parameterize each curve in the form $\vec{r}(t) = (x(t), y(t), z(t))$.
- b) (3 points) Set up the integral for the arc length of one of the curves.
- c) (3 points) What is the arc length of this curve?

Problem 8) (10 points)

- a) (6 points) Find the curvature $\kappa(t)$ of the space curve $\vec{r}(t) = \langle -\cos(t), \sin(t), -2t \rangle$ at the point $\vec{r}(0)$.
- b) (4 points) Find the curvature $\kappa(t)$ of the space curve $\vec{r}(t) = \langle -\cos(5t), \sin(5t), -10t \rangle$ at the point $\vec{r}(0)$.

Hint. Use one of the two formulas for the curvature

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3},$$

where $\vec{T}(t) = \vec{r}'(t)/|\vec{r}'(t)|$. The curvatures in b) can be derived from the curvature in a). There is no need to redo the calculation in b) if you give a proper justification.

Problem 9) (10 points)

For each of the following, fill in the blank with $<$ (less than), $>$ (greater than), or $=$ (equal).

Justify your answer completely.

- | | |
|---|--|
| <p>1. The arc length of the curve parameterized by $\vec{f}(t) = \langle \cos 2t, 0, \sin 2t \rangle$, $0 \leq t \leq \pi$. _____</p> | <p>The arc length of the curve parameterized by $\vec{g}(u) = \langle 3, 2 \cos u^2, 2 \sin u^2 \rangle$, $0 \leq u \leq \sqrt{\pi}$.</p> |
| <p>2. The arc length of the curve parameterized by $\vec{f}(t) = \langle t^2, 2 \cos t, 2 \sin t \rangle$, $0 \leq t \leq 2\pi$. _____</p> | <p>The arc length of the curve parameterized by $\vec{g}(u) = \langle u^4, 2 \cos u^2, 2 \sin u^2 \rangle$, $0 \leq u \leq 2\pi$.</p> |
| <p>3. The arc length of the curve parameterized by $\vec{f}(t) = \langle 1 + 3t^2, 2 - t^2, 5 + 2t^2 \rangle$, $0 \leq t \leq 1$. _____</p> | <p>The arc length of the curve parameterized by $\vec{g}(u) = \langle \frac{1}{2}u^2, u, \frac{2\sqrt{2}}{3}u^{3/2} \rangle$, $0 \leq u \leq 2$.</p> |
| <p>4. The arc length of the curve parameterized by $\vec{f}(t) = \langle \sin t, \cos t, t \rangle$, $1 \leq t \leq 5$. _____</p> | <p>The arc length of the curve parameterized by $\vec{g}(u) = \langle u \sin u, u \cos u, u \rangle$, $1 \leq u \leq 5$.</p> |

Problem 10) (10 points)

Given the plane $x + y + z = 6$ containing the point $P = (2, 2, 2)$. Given is also a second point $Q = (3, -2, 2)$.

- a) (5 points) Find the equation $ax + by + cz = d$ for the plane through P and Q which is perpendicular to the plane $x + y + z = 6$.
- b) (5 points) Find the symmetric equation for the intersection of these two planes.

