

Name:

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MWF 10 Adrian Zahariuc
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MWF 11 Peter Garfield
MWF 11 Matthew Woolf
MWF 12 Charmaine Sia
MWF 12 Steve Wang
MWF 14 Mike Hopkins
TTH 10 Oliver Knill
TTH 10 Francesco Cavazzani
TTH 11:30 Kate Penner
TTH 11:30 Francesco Cavazzani

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work.** Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False (TF) questions (20 points)
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Mark for each of the 20 questions the correct letter. No justifications are needed.

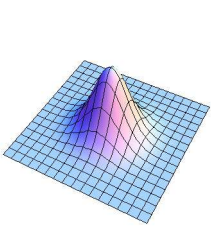
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|-----|---|--|
| 1) | ☐ T ☐ F | If two planes $ax + by + cz = d$ and $Ax + By + Cz = D$ are parallel then $a = A, b = B$, and $c = C$. |
| 2) | ☐ T ☐ F | The point $(x, y, z) = (1, 1, \sqrt{2})$ has the spherical coordinates $(\rho, \theta, \phi) = (2, \pi/4, \pi/4)$. |
| 3) | ☐ T ☐ F | Every point on the parametric curve $\vec{r}(t) = \langle t, t^2, -t \rangle$ lies on the surface $xz + y = 0$. |
| 4) | ☐ T ☐ F | The two surfaces $f(x, y, z) = 3$ and $f(x, y, z) = 5$ of the function $f(x, y, z) = 2x^2 + y^3 + z^4$ do not intersect at any point in space. |
| 5) | ☐ T ☐ F | $\vec{u} \times \vec{i}$ and $\vec{u} \times \vec{j}$ are perpendicular for all vectors $\vec{u}$. |
| 6) | ☐ T ☐ F | If $\vec{u}$ and $\vec{v}$ are parallel (remember that this means $\vec{u} = \lambda \vec{v}$ for some real λ) then $\vec{u} \cdot \vec{v} \geq \vec{u} \times \vec{v} $. |
| 7) | ☐ T ☐ F | If a surface has the property that all intersections with the planes $y = \text{constant}$ are straight lines, then the surface is a plane. |
| 8) | ☐ T ☐ F | For any non-zero vectors $\vec{u}$ and $\vec{w}$, we must have $\text{Proj}_{\vec{u}} \vec{w} = -\text{Proj}_{\vec{w}} \vec{u}$. |
| 9) | ☐ T ☐ F | In the parametric surface $\vec{r}(s, t) = \langle \sqrt{1+e^t} \cos(s), \sqrt{1+e^t} \sin(s), t \rangle$ the grid curves with constant s are ellipses. |
| 10) | ☐ T ☐ F | There is a vector $\vec{v}$ with the property that $\vec{v} \times \langle 1, 1, 1 \rangle = \langle 0, 0, 1 \rangle$. |
| 11) | ☐ T ☐ F | We can assign a value $f(0, 0)$ such that the function $f(x, y) = (x^3 + y^3)/(x^2 + y^2)$ is continuous at $(0, 0)$. |
| 12) | ☐ T ☐ F | The curvature of a curves $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ and $\vec{R}(t) = \langle t^2, t^4, t^6 \rangle$ are the same at $t = 1$. |
| 13) | ☐ T ☐ F | The curve given in spherical coordinates as $\phi = \pi/2, \rho = \pi/2$ is a circle. |
| 14) | ☐ T ☐ F | Two nonparallel planes with normal vectors $\vec{n}, \vec{m}$ intersect in a line parallel to $\vec{n} \times \vec{m}$. |
| 15) | ☐ T ☐ F | If $f(x, y) = x^3/3 - y^2$, then the graph of the function $f(x, y)$ is called an elliptic paraboloid. |
| 16) | ☐ T ☐ F | The equation $\rho \cos(\theta) \sin(\phi) = 2$ in spherical coordinates defines a plane. |
| 17) | ☐ T ☐ F | The vector $\langle 3, -2 \rangle$ in the two dimensional plane is perpendicular to the line $3x - 2y = 7$. |
| 18) | ☐ T ☐ F | The volume of the parallelepiped spanned by the vectors $\langle 1, 0, 0 \rangle, \langle 0, 2, 0 \rangle$ and $\langle 1, 1, 1 \rangle$ is 2. |
| 19) | ☐ T ☐ F | If $\vec{r}(t)$ is a curve and $ \vec{r}'(t) > 0$ and $ \vec{T}' > 0$, we have $\vec{T}(t) \cdot (\vec{N}(t) \times \vec{B}(t)) = 1$. |
| 20) | ☐ T ☐ F | The arc lengths of $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ and $\vec{R}(t) = \langle t^2, t^4, t^6 \rangle$ are the same for $0 \leq t \leq 1$. |

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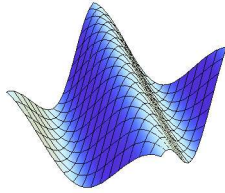
Total

Problem 2) (10 points)

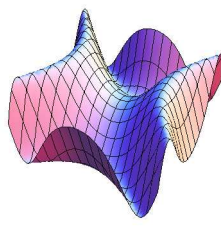
a) (2 points) Match the graphs $z = f(x, y)$ with the functions. Enter O, if there is no match. In each of the problems a) - d), each entry O,I,II,III appears exactly once.



I



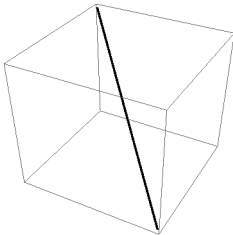
II



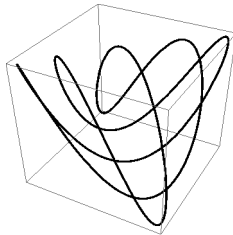
III

Function $f(x, y) =$	O,I,II or III
$e^{-x^2-y^2}$	
$\cos(x + y)$	
$\sin(x^2 - y^2)$	
$x^4 + y^4$	

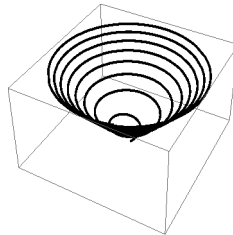
b) (3 points) Match the space curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.



I



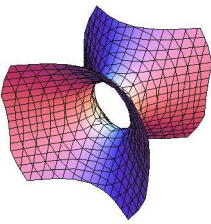
II



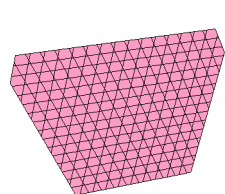
III

Parametrization $\vec{r}(t) =$	O, I,II,III
$\langle 1 + t, 1 - t, t \rangle$	
$\langle t \cos(t^2), t \sin(t^2), t \rangle$	
$\langle t, t, \sin(t^3) \rangle$	
$\langle \cos(3t), \sin(2t), \sin(5t) \rangle$	

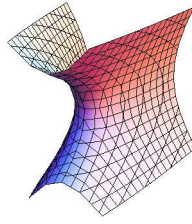
c) (2 points) Match the functions g with the level surface $g(x, y, z) = 1$. Enter O, where no match.



I



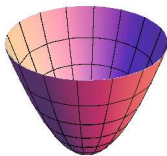
II



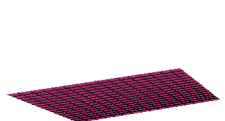
III

$g(x, y, z) =$	O, I,II,III
$(x - 1)^2 - y^2 + z^2 = 1$	
$(x - 1)^2 + y + z^2 = 1$	
$(x - 1) + y + z = 1$	
$(x - 1)^2 - y - z^2 = 1$	

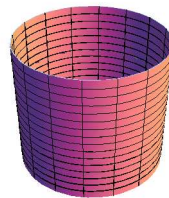
d) (3 points) Match the surface with the parametrization. Enter O, where no match.



I



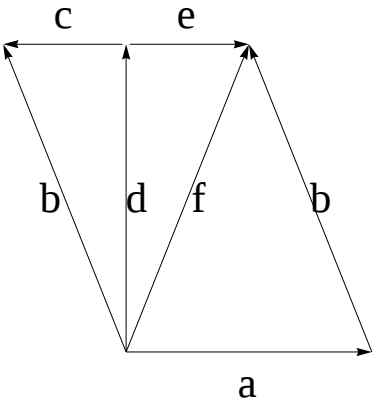
II



III

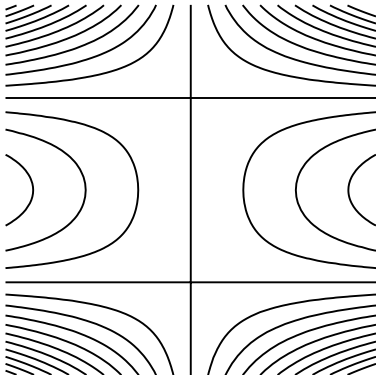
Parametrization $\vec{r}(s, t) =$	O,I,II,III
$\langle s \cos(t), s \sin(t), s^2 \rangle$	
$\langle t - 1, s, s + t \rangle$	
$\langle \cos(t), \sin(t), s \rangle$	
$\langle s \cos(t), s \sin(t), s^2 \sin(t) \rangle$	

a) (7 points) Each of the vectors $a, b, c, d, e, f, 0$ (written without arrows for clarity) will appear in the blanks exactly once. As the picture indicates, you know $d \cdot e = d \cdot c = 0$.

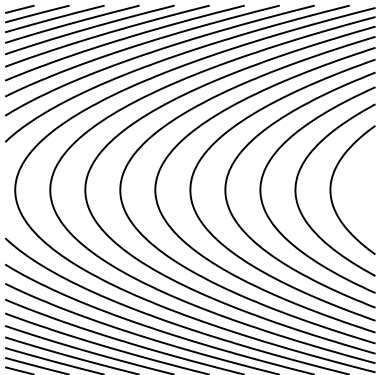


the vector	is equal to
$\text{Proj}_d f$	
$f - d$	
$-2c$	
$d - c$	
$-e$	
$\text{Proj}_d e$	
$d + c$	

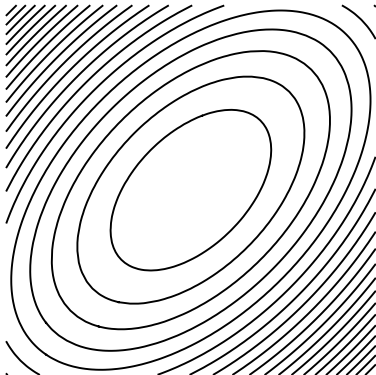
b) (3 points) Match the contour maps with the functions



I



II



III

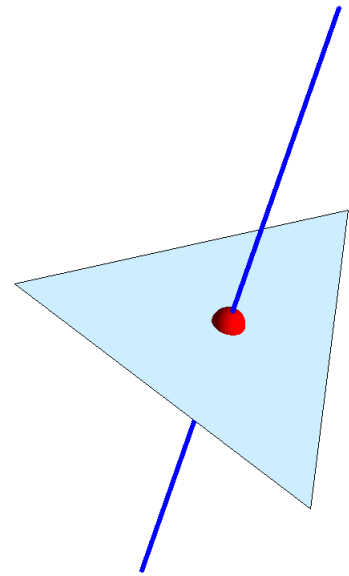
Function $f(x, y) =$	Enter O,I,II or III
$y - x$	
$(y^2 - 1)x$	
$y^2 + x^2 - xy$	
$y^2 - x$	

Problem 4) (10 points)

a) (4 points) The **center** of the triangle $A = (3, 2, 1), B = (1, 1, 1), C = (2, 0, 4)$ is the point $P = (A + B + C)/3 = (2, 1, 2)$. Find the line L perpendicular to the plane which contains A, B, C and which goes through P .

b) (3 points) Find the equation of the plane through A, B, C .

c) (3 points) Find the area of the triangle ABC .



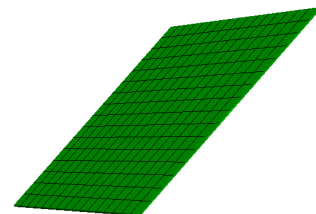
Problem 5) (10 points)

Complete the parametrizations:

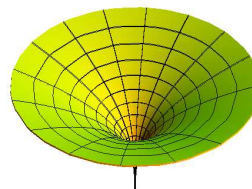
a) (3 points) $\vec{r}(u, v) = \langle 2 + 3 \cos(u) \sin(v), 3 + \sin(u) \sin(v), \boxed{} \rangle$ parametrizes the ellipsoid $(x - 2)^2/9 + (y - 3)^2 + (z - 5)^2/16 = 1$.



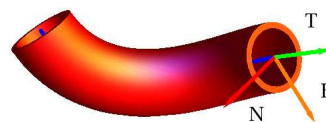
b) (2 points) $\vec{r}(u, v) = \langle u, v, \boxed{} \rangle$ parametrizes the plane $x + y + z = 1$.



c) (3 points) $\vec{r}(u, v) = \langle v^3 \cos(u), \boxed{}, v \rangle$
 parametrizes the surface of revolution $x^2 + y^2 = z^6$.



d) (2 points) $\vec{r}(u, v) = \vec{r}(v) + \cos(u)\vec{N}(v) + \sin(u)\boxed{}$ parametrizes a tube around a curve $\vec{r}(v)$ which has unit tangent vector $\vec{T}(v)$, normal vector $\vec{N}(v)$ and binormal vector $\vec{B}(v)$.



Problem 6) (10 points)

We look at the parametrized curve

$$\vec{r}(t) = \left\langle \frac{t^3}{3} - t, t^2 - 1, 0 \right\rangle$$

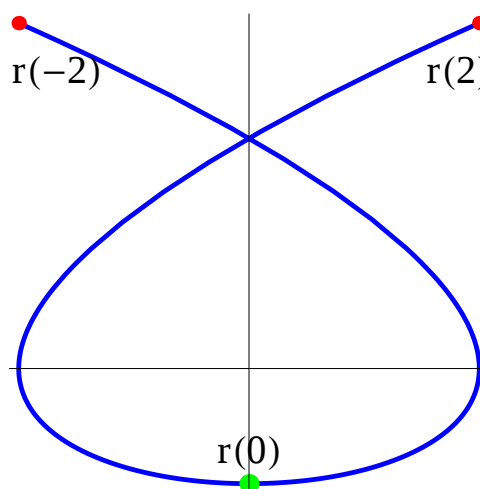
whose image you see in the picture showing it in the xy plane for $-2 \leq t \leq 2$.

a) (3 points) Find the velocity $\vec{r}'(t)$, the acceleration $\vec{r}''(t)$ and speed $|\vec{r}'(t)|$.

b) (2 points) Evaluate this at $t = 0$ to get $\vec{r}'(0)$, $\vec{r}''(0)$ and $|\vec{r}'(0)|$.

c) (2 points) Find the curvature $|\vec{r}'(0) \times \vec{r}''(0)| / |\vec{r}'(0)|^3$ at $(0, -1, 0)$.

d) (3 points) Find the arc length of the curve $\vec{r}(t)$ from $-2 \leq t \leq 2$.

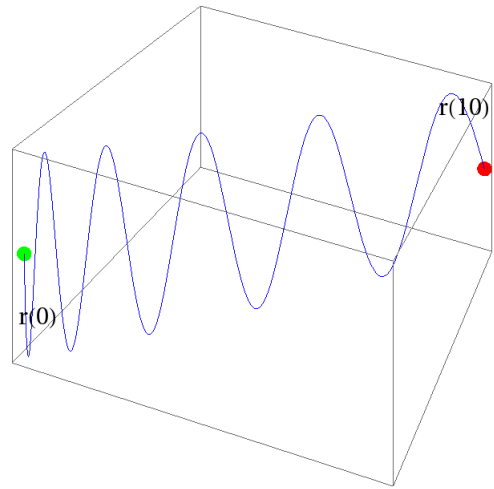


Problem 7) (10 points)

a) (4 points) We know $\vec{r}''(t) = \langle 1, 2, \pi^2 \sin(\pi t) \rangle$ and the initial velocity $\vec{r}'(0) = \langle 1, 0, -\pi \rangle$. Find $\vec{r}'(t)$.

b) (3 points) Assume we know also $\vec{r}(0) = \langle 0, 0, 10 \rangle$. Find $\vec{r}(10)$.

c) (3 points) What is the projection of $\vec{r}'(10)$ onto $\langle 1, 1, 0 \rangle$?



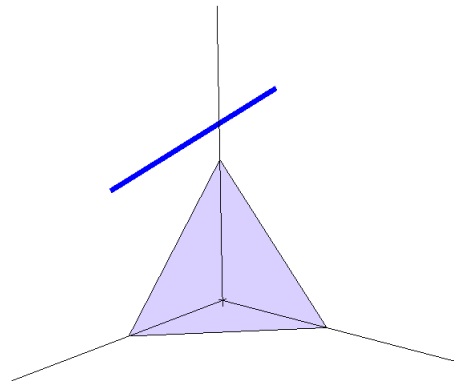
Problem 8) (10 points)

a) (5 points) Find the distance between the plane $x + y + z = 1$ and the line

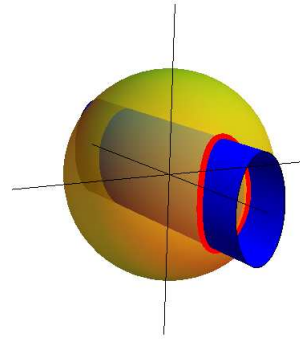
$$x - 1 = \frac{(y - 1)}{-2} = z - 1$$

which is parallel to the plane.

(You do not have to check that it is parallel).



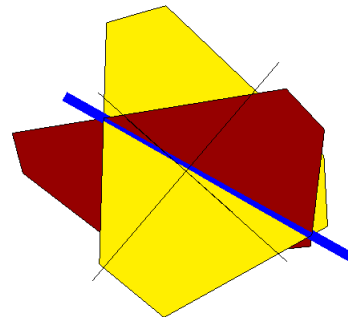
b) (5 points) The intersection of the cylinder $4x^2 + z^2 = 1$ with the sphere centered at $(0, 0, 0)$ with radius $\rho = \sqrt{2}$ cuts out two curves. Parametrize the curve which contains the point $(0, 1, 1)$.



Problem 9) (10 points)

a) (5 points) Find a parametrization of the intersection line L of the two planes

$$\begin{aligned} 2x - 2y + z &= 1, \\ x + y + z &= 1. \end{aligned}$$



b) (5 points) Find the symmetric equation for the line M parallel to the line L computed in a) which passes through $(1, 2, 3)$.

Problem 10) (10 points)

a) (5 points) What is the area of the triangle through the points $A = (1, 1, 1)$ and $B = (0, 1, 0)$ and $C = (1, 2, 4)$.

b) (5 points) Find the volume of the prism which has the triangle T as base as well as a by $\vec{v} = \langle 0, 1, 1 \rangle$ translated triangle as roof.

