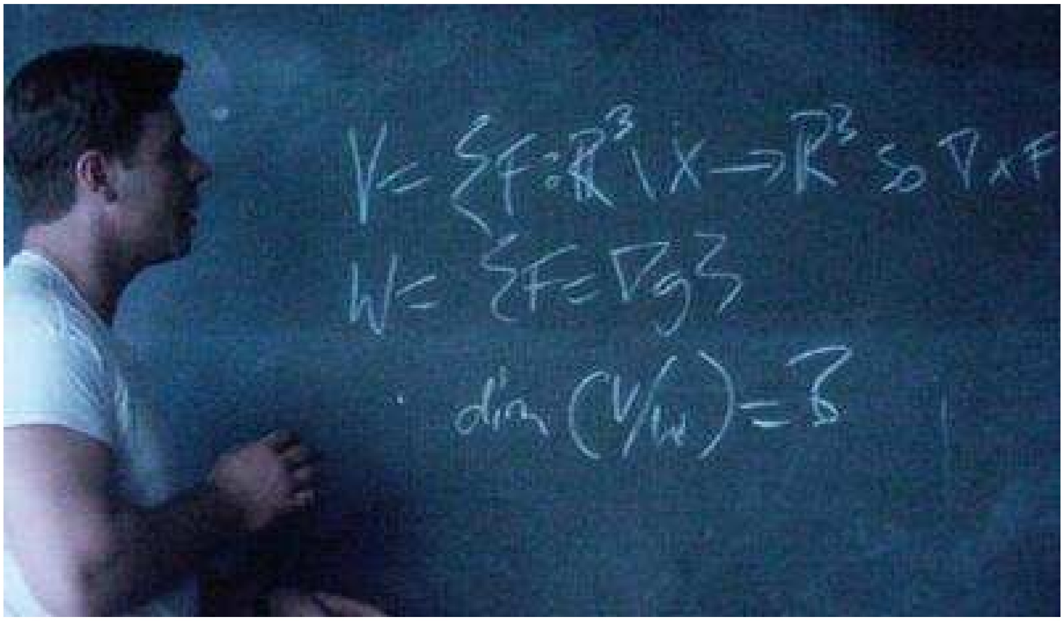
ICE: NASH'S PROBLEM **Math 21a**

In this ICE, you solve Nash's problem, he gave to a multivariable calculus class. Remember Nash saying in the movie "A beautiful mind":

> "This problem here will take some of you many months to solve, for others amoung you it might take a life time".

You will solve it here in 10 minutes ...



NASH's PROBLEM. Find a subset X of $\mathbf{R}^3$ with the property that if V is the set of vector fields F on $\mathbf{R}^3 \setminus X$ which satisfy $\text{curl}(F) = 0$ and W is the set of vector fields F which are conservative: $F = \nabla f$. Then, the space V/W should be 8 dimensional.

Remark. The meaning of the last sentence means that there should be 8 vectorfields F_i which are not gradient fields and which have vanishing curl outside X. Furthermore, you should not be able to write any of the 8 vectorfields as a sum of multiples of the other 7 vector fields.

Here is a two dimensional version of the problem:

2D VERSION OF NASH's PROBLEM.

If $X = \{0\}$ and V is the set of vector fields F on $\mathbf{R}^2 \setminus X$ which satisfy $\text{curl}(F) = 0$ and W is the set of vector fields F which are gradient fields, then $\dim(V/W) = 1$.

The vector field F is $F(x, y) = (-y/(x^2 + y^2), x/(x^2 + y^2))$ has vanishing curl in $\mathbf{R}^2 \setminus X$ but the line integral around the origin is 2π. It is not a gradient field.
Now: What set X would you have to take to get $\dim(V/W) = 8$?

The SOLUTION OF THE 3D VERSION OF NASH's PROBLEM can be obtained directly from the solution of the 2D version. How?