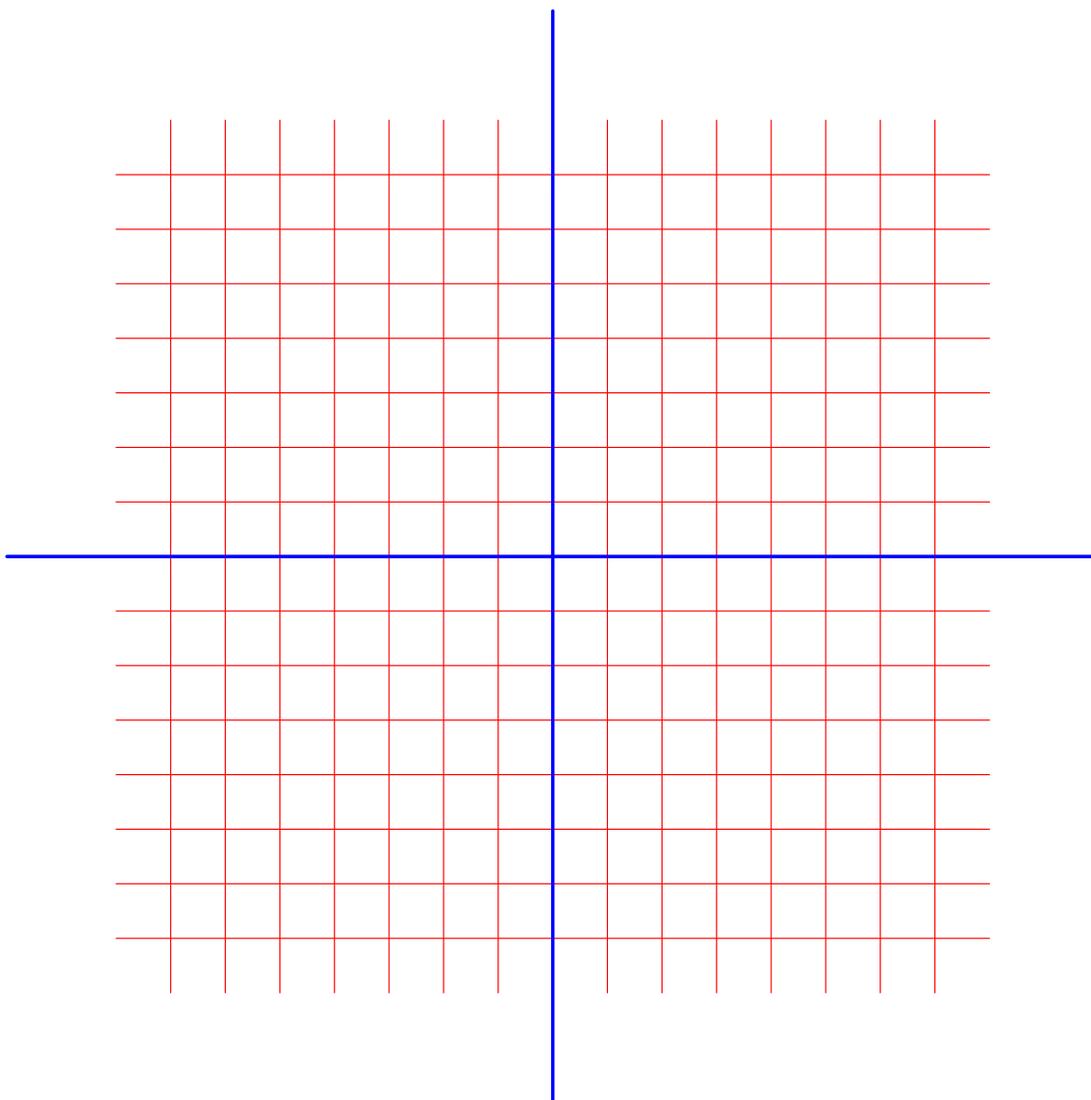


The **threefoil knot** is the space curve

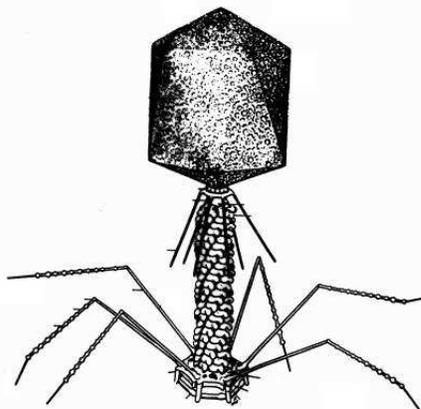
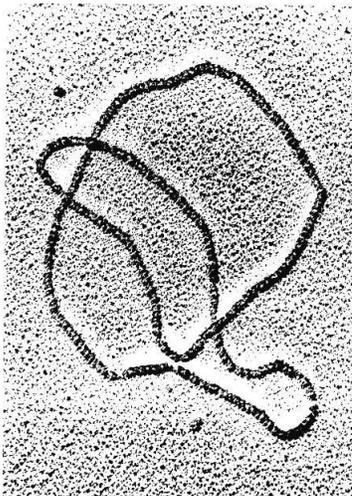
$$\vec{r}(t) = ((2 + \cos(3t/2)) \cos(t), (2 + \cos(3t/2)) \sin(t), \sin(3t/2))$$

- 1) Find an interval $[a, b]$ on which $\vec{r}(t)$ parameterizes a closed curve in space. A closed curve in space is called a **knot**. These objects are not only interesting in mathematics or physics. For example, DNA of bacteria and some proteins form knots (see the back of the page).
- 2) Verify that the projection $\vec{r}(t) = \langle x(t), y(t) \rangle$ of this curve onto the xy -plane is in polar coordinates by $\vec{r}(t) = 2 + \cos(3t/2), \theta(t) = t$.
- 3) Sketch this curve.



Indicate at the crossings, which part of the curve is above the other.

- 4) Calculate the velocity vector $\vec{r}'(t)$ of $\vec{r}(t)$ at $t = 0$. If this vector is $\vec{v} = \langle a, b, c \rangle$, then $\langle a, b \rangle$ is its projection on the xy plane. Draw this vector $\langle a, b \rangle$ in the above xy -projection of the knot.



Communications to the Editor

A Real Knot in Protein

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It is well-known that circular DNAs exhibit a rich variety of knotted structures.¹ Recent surveys of the X-ray structures deposited in the Brookhaven Protein Data Bank have revealed the presence of pseudolinks and pseudoknots in protein structures caused by formation of disulfide bonds and metal coordination bonds.² However, there has been no report so far of knots in native proteins or polypeptides.³ We now report our finding of a linear knot in the structure of (S)-adenosylmethionine synthetase (MAT) recently determined in our laboratory.³

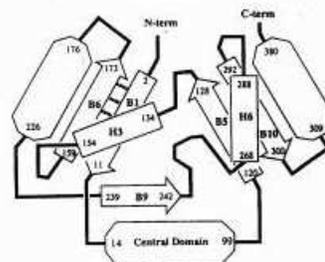


Figure 1. Schematic drawing of the unusual knot structure of the polypeptide chain of MAT. The knot is formed by passage of the B9 β -strand leading to the C-terminus through a loop formed by the sequence B1 \rightarrow [central domain] \rightarrow B5 \rightarrow B3 \rightarrow B6. The rectangles, arrows, and elongated octagons represent α -helices, β -strands, and portion of domains, respectively. The numbers at both ends are the start and end of the amino acid residue numbers. Hydrogen bonds between B1 and B6 β -strands are shown by dotted lines.



Chart 1. Ribbon Presentation of the MAT Subunit*

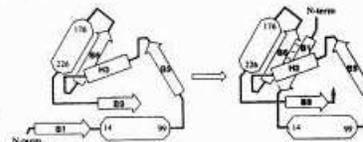
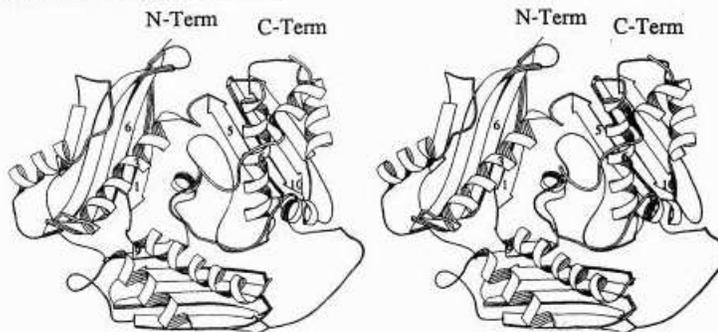


Figure 2. Hypothesis for biosynthetic knot formation. The N-terminal region (residues 1-11) locates near the central domain until after the synthesis of the B9 β -strand (residue 242). After synthesis of the B9 β -strand region, the N-terminal region moves into the closed loop, and the B1 β -strand (residues 2-11) forms antiparallel β -sheet hydrogen bonds with the B6 β -strand shown by dotted lines.

For more information on DNA knots, see the MSRI talk by DeWitt Summers:
<http://www.msri.org/publications/ln/msri/2000/molbio/summers/1/>