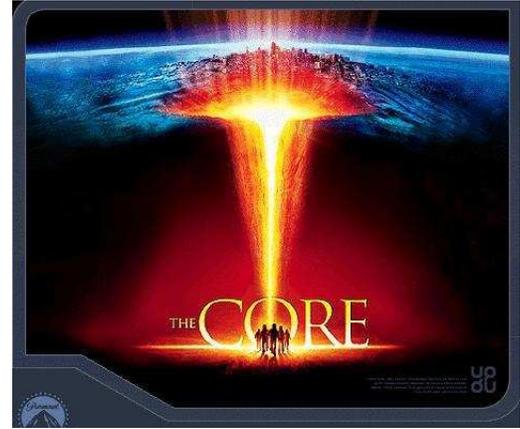




Since the middle ages, people were interested in the gravity of earth. Originally, the motivation was to figure out how life would be like in hell ... Nowadays, the interest has sparked by science fiction movies like "the core", where people travel to the core of the earth.



GRAVITY INSIDE THE EARTH. How much do we weight deep in earth at radius r from the center of the earth? (Relevant in the movie "The core") The law of gravity can be formulated as

$$\operatorname{div}(F) = 4\pi\rho$$

where ρ is the mass density.

We assume that the earth is a ball of radius R . By rotational symmetry, the gravitational force is normal to the surface:

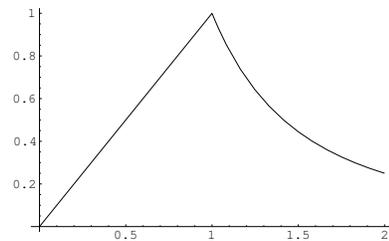
$$F(x) = F(r)x/\|x\|.$$

The flux of F through a ball of radius r is

$$\int \int_{S_r} F(x) \cdot dS = 4\pi r^2 F(r)$$

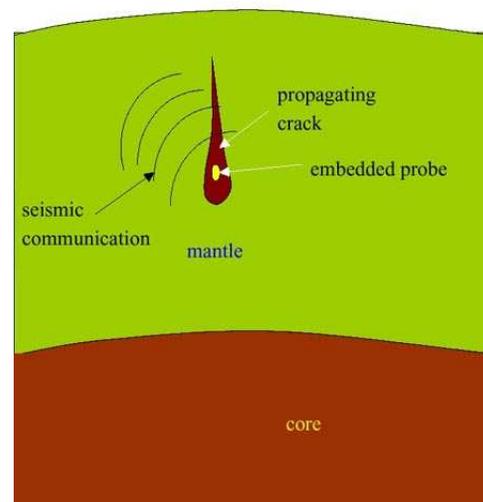
By the **divergence theorem**, this is $4\pi M_r = 4\pi \int \int \int_{B_r} \rho(x) dV$, where M_r is the mass of the material inside S_r . We have $(4\pi)^2 \rho r^3 / 3 = 4\pi r^2 F(r)$ for $r < R$ and $(4\pi)^2 \rho R^3 / 3 = 4\pi r^2 F(r)$ for $r \geq R$.

Inside the earth, the gravitational force $F(r) = 4\pi\rho r/3$. Outside the earth, it satisfies $F(r) = M/r^2$ with $M = 4\pi R^3 \rho/3$.



We have seen above, how the gravitational field behaves inside a homogeneous body. This would have been a difficult computation without the divergence theorem.

Since we know more about the interior of other stars than the interior of our earth, people still think about sending probes to the earth center. One of the first proposals was by Zwicky who wanted to use tunnels through the earth for travel purposes. One of the recent proposals is to send liquid metal down a crack and let this propagate.



A nuclear explosion or earthquake might reveal the Earth's innards

MICHAEL HOPKIN

Abstract: Cracking idea gets to the core Astronomer says time is ripe for an unmanned journey to the centre of the Earth.

We have sent probes to Mars, the outer planets and even beyond the farthest reaches of the Solar System - but never to the centre of our own world. Now a planetary scientist has a radical plan to redress the balance.

Outer space is trillions upon trillions of times bigger than the Earth's interior, and yet we know more about what's out there than we do about what's under our feet, says David Stevenson of the California Institute of Technology in Pasadena.

The Earth's core is "crammed with interesting stuff", he says. For example, swirling currents of liquid metal 3,000-5,000 kilometers below the surface are thought to generate the planet's magnetic field.

So Stevenson suggests we blast a crack in the Earth's surface and pour in thousands of tonnes of molten iron. The iron-filled crack would burrow downwards, closing up behind itself as it descends, and reaching the bowels of the planet in about a week.

Concealed in this seething blob of iron would be a grapefruit-sized probe containing instruments to measure temperature, electrical conductivity and chemical composition. The device would send data as sound waves to a surface detector, which would filter the signal from the Earth's natural rumblings.

Starting the crack would be the tricky part, says Stevenson. He calculates that it would take a blast equivalent to several megatonnes of TNT, an earthquake of magnitude 7 on the Richter scale, or a nuclear device such as those already possessed by many nations.

Getting hold of the iron would also be an impressive logistical feat - Stevenson reckons that the experiment would need to commandeer all of the world's iron foundries for anything from an hour to a week. He is confident, however, that the molten reservoir's sheer size would stop it solidifying before it is poured into the ground.

It's an interesting proposition, comments Allan Rubin, who studies the Earth's structure at Princeton University in New Haven, Connecticut, but it does make some rather blithe assumptions. "We don't even know for definite that the lower mantle can fracture," he points out.

Stevenson claims that the cost of his plan would be small beans compared with the riches lavished on space exploration. But he offers prospective investors no guarantee of success: "Frankly, I would be surprised if this really works," he admits.

References

1. Stevenson, D. J. Mission to Earth's core - a modest proposal. *Nature*, 423, 239 - 240, (2003).