Lecture 10: Worksheet

The chain rule

The rule$(f(g(x))' = f'(g(x))g'(x)$ is called the **chain rule**.
For example, the derivative of $\sin(\log(x))$ is $\cos(\log(x))/x$.
We have also seen that we can compute the derivative of inverse functions using the chain rule.

1 Find the derivative of $\sqrt{1+x^2}$ using the chain rule

2 Find the derivative of $\sin^3(x)$ using the product rule.

3 Find the derivative of $\sin^3(x)$ using the chain rule.

4 Find the derivative of $\tan(\sin(x))$.

5 Find the derivative of $\sin(\cos(\exp(x)))$.

6 Find the derivative of $\arcsin(x)$ using the chain rule.

A lovely application of the chain rule

The **Valentine equation** (from last Friday). $(x^2 + y^2 - 1)^3 - x^2 y^3 = 0$ relates x with y, but we can not write the curve as a graph of a function $y = g(x)$. Extracting y or x is difficult. The set of points satisfying the equation looks like a heart. You can check that $(1, 1)$ satisfies the Valentine equation. Near it, the curve looks like the graph of a function $g(x)$. Lets fill that in and look at the function

$$f(x) = (x^2 + g(x)^2 - 1)^3 - x^2 g(x)^3$$

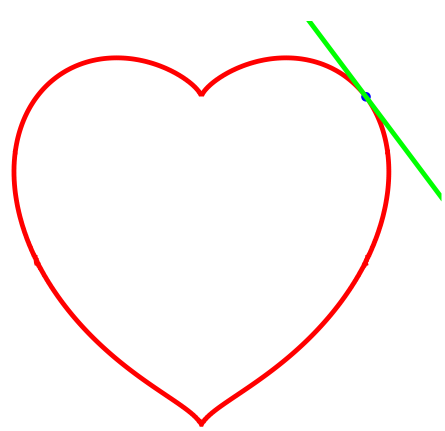
The key is that $f(x)$ is actually zero and if we take the derivative, then we get zero too. Using the chain rule, we can take the derivative

$$f'(x) = 3(x^2+g(x)^2-1)(2x+2g(x)g'(x))-2xg(x)^3-x^2 3g(x)^2 g'(x) = 0$$

We can now solve solve for g'

$$g'(x) = -\frac{3(x^2 + g(x)^2 - 1)2x - 2xg(x)^3}{3(x^2 + g(x)^2 - 1)2g(x) - 3x^2 g(x)^2} .$$

Filling in $x = 1, g(x) = 1$, we see this is $-4/3$. We have computed the slope of g without knowing g. Magic!



We will come back to this application of the chain rule later in the course.