3/1/2012: First Midterm Exam

Your Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- Except for multiple choice problems, give computations.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) TF questions (20 points) No justifications are needed.

1)	T	F	1 is the only root of the log function on the interval $(0, \infty)$.
2)	T	F	$\exp(\log(5)) = 5$, if log is the natural log and $\exp(x) = e^x$ is the exponential function.
3)	T	F	The function $\cos(x)+\sin(x)+x^2$ is continuous everywhere on the real axes.
4)	T	F	The function $\sec(x) = 1/\cos(x)$ is the inverse of the function $\cos(x)$.
5)	T	F	The Newton method allows to find the roots of any continuous function.
6)	T	F	$\sin(3\pi/2) = -1$.
7)	T	F	If a function f is continuous on $[0, \infty)$, then it has a global maximum on this interval.
8)	T	F	The reciprocal rule assures that $d/dx(1/g(x)) = -1/g(x)^2$.
9)	T	F	If $f(0) = g(0) = f'(0) = g'(0) = 0$ and $g''(0) = f''(0) = 1$, then $\lim_{x \to 0}(f(x)/g(x)) = 1$
10)	T	F	An inflection point is a point, where the function $f''(x)$ changes sign.
11)	T	F	If $f''(3) > 0$ then f is concave up at $x = 3$.
12)	T	F	The intermediate value theorem assures that a continuous function has a maximum on a finite interval.
13)	T	F	We can find a value b and define $f(0) = b$ such that the function $f(x) = (x^6 - 1)/(x^3 - 1)$ is continuous everywhere.
14)	T	F	Single roots of the second derivative function f'' are inflection points.
15)	T	F	If the second derivative $f''(x)$ is negative and $f'(x) = 0$ then f has a local maximum at x.
16)	T	F	The function $f(x) = [x]^3 = x(x + h)(x + 2h)$ satisfies $Df(x) = 3[x]^2 = 4x(x + h)$, where $Df(x) = [f(x + h) - f(x)]/h$.
17)	T	F	The quotient rule is $d/dx(f/g) = (fg' - f'g)/g^2$.
18)	T	F	The chain rule assures that $d/dx f(g(x)) = f'(g(x))f'(x)$.
19)	T	F	With $Df(x) = f(x + 1) - f(x)$, we have $D2^x = 2^x$.
20)	T	F	Hôpital's rule applied to the function $f(x) = \text{sinc}(x) = \sin(x)/x$ gives us the fundamental theorem of trigonometry.

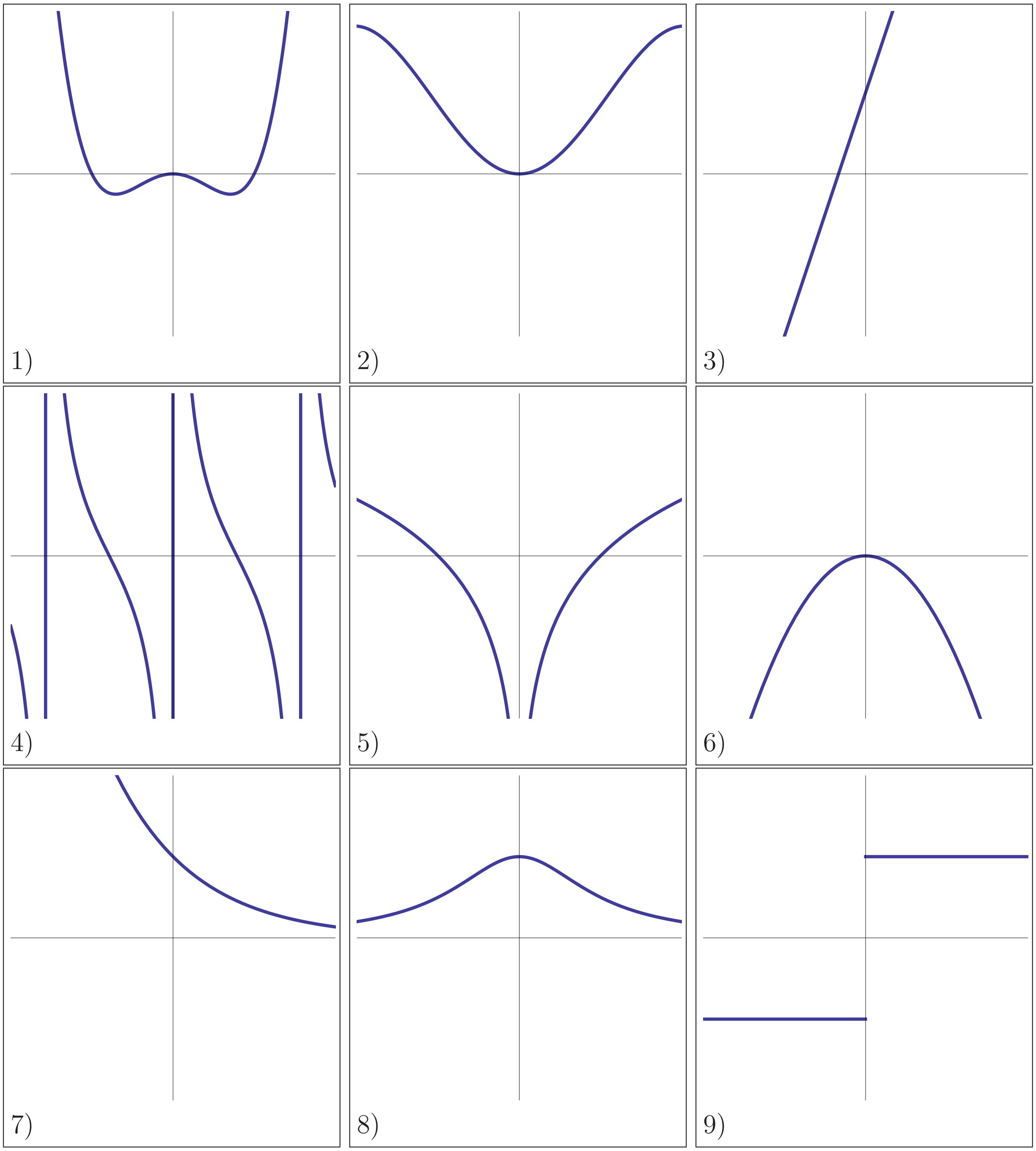
Problem 2) Matching problem (10 points) No justifications are needed.

Match the functions with the graphs.

Function	Enter 1-9
$1/(1+x^2)$	
$\cot(2x)$	
$3x+1$	

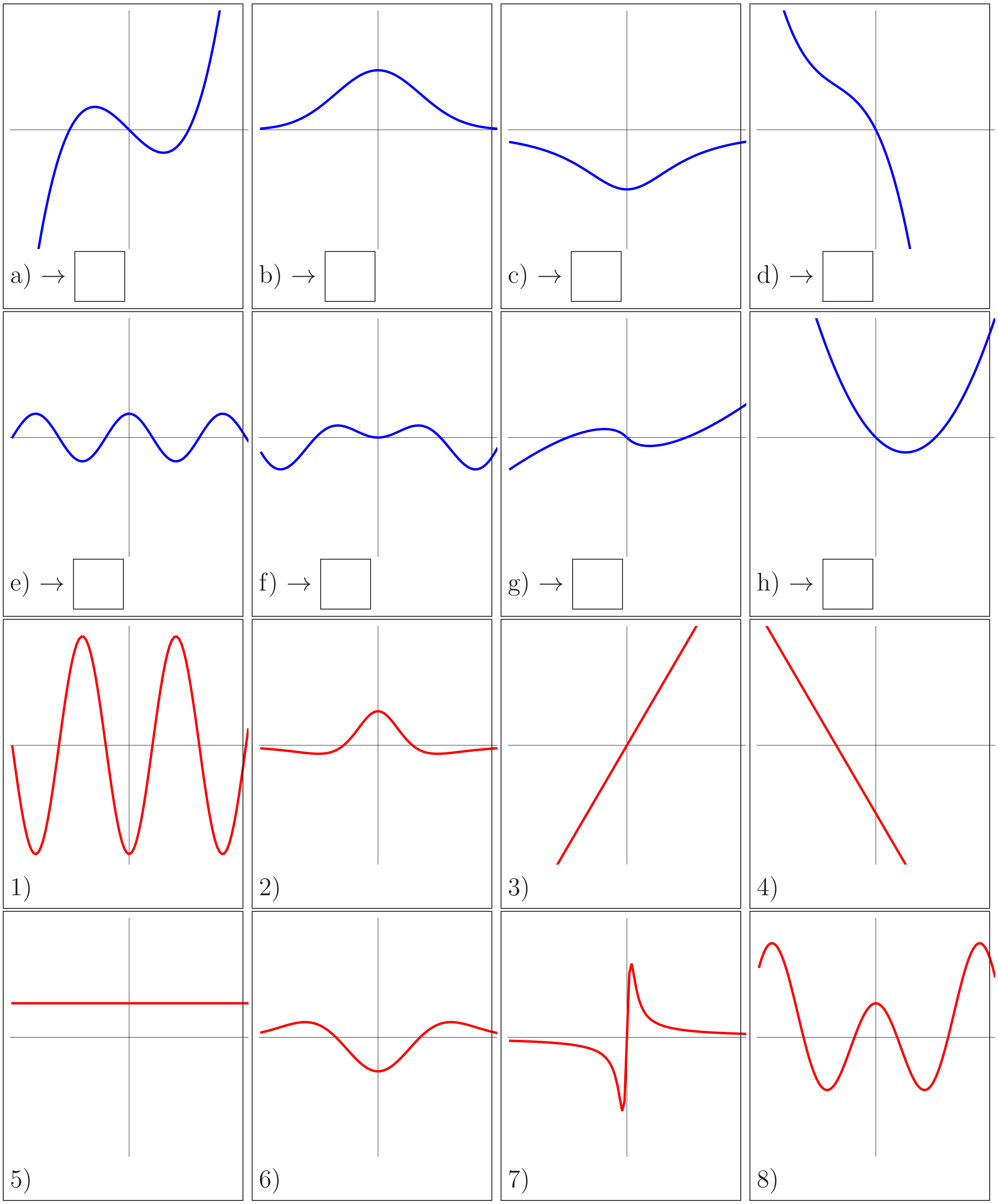
Function	Enter 1-9
$x\sin(x)$	
$\exp(-x)$	
$\log(\vert x\vert)$	

Function	Enter 1-9
$\mathrm{sign}(x)$	
x^4-x^2	
$-x^2$	



Problem 3) Matching problem (10 points) No justifications are needed.

Match the graph of the functions f in $a)-h)$ with the second derivatives f'' in 1)-8).



Problem 4) Continuity (10 points)

Decide whether the function can be healed at the given point in order to be continuous everywhere on the real line. If the function can be extended to a continuous function, give the value at the point.

a) (2 points) $f(x) = \frac{(x^3-8)}{(x-2)}$, at $x = 2$

b) (2 points) $f(x) = \sin(\sin(1/x)) - \tan(x)$, at $x = 0$

c) (2 points) $f(x) = \frac{\cos(x)-1}{x^2}$, at $x = 0$

d) (2 points) $f(x) = (\exp(x) - 1)/(\exp(5x) - 1)$, at $x = 0$

e) (2 points) $f(x) = \frac{(x-1)}{x}$, at $x = 0$

Problem 5) Chain rule (10 points)

In the following cases, we pretend not to know the formula for the derivative of log or arctan and again recover it using the chain rule.

b) (2 points) Rederive the derivative of the square root function $\text{sqrt}(x) = \sqrt{x}$ by differentiating

$$(\text{sqrt}(x))^2 = x$$

and solving for $\text{sqrt}'(x)$.

b) (4 points) Rederive the derivative of the logarithm function $\log(x)$ by differentiating

$$\exp(\log(x)) = x$$

and solving for $\log'(x)$.

c) (4 points) Rederive the formula for the derivative of the arctan function $\arctan(x)$ by differentiating the identity

$$\tan(\arctan(x)) = x$$

and using $1 + \tan^2(x) = 1/\cos^2(x)$ to solve for $\arctan'(x)$.

Problem 6) Derivatives (10 points)

Find the derivatives of the following functions:

a) (2 points) $f(x) = \frac{5\sin(x^6)}{x}$ for $x > 0$

b) (2 points) $f(x) = \tan(x^2) + \cot(x^2)$ for $x > 0$

c) (2 points) $f(x) = \frac{1}{x} + \log(x^2)$ for $x > 0$

d) (2 points) $f(x) = x^6 + \sin(x^4)\log(x)$ for $x > 0$

e) (2 points) $f(x) = \log(\log(x))$ for $x > 1$

Problem 7) Limits (10 points)

Find the limits $\lim_{x \to 0} f(x)$ for the following functions f at $x = 0$ or state that the limit does not exist. State the tools you are using.

a) (2 points) $f(x) = x^2 + x + \sin(1 - \cos(x))$

b) (2 points) $f(x) = \frac{x^3}{\sin(x^3)}$

c) (2 points) $f(x) = x^3/\sin(x)^2$

d) (2 points) $f(x) = x^3 + \text{sign}(x)$

e) (2 points) $f(x) = \cos(x^4) + \cos(\frac{1}{x})$

Problem 8) Extrema (10 points)

In the following problem you can ignore the story if you like and proceed straight go to the question:

Story: a cone shaped lamp designed in 1995 by **Verner Panton** needs to have volume $\pi r^2 h = \pi$ to be safe. To minimize the surface area $A = \pi r \sqrt{h^2 + r^2}$, we minimize the square A^2 and so $\pi^2 r^2 (h^2 + r^2)$. From the volume assumption, we get $r^2 = 1/h$ so that we have to minimize $\pi^2/h(h^2 + 1/h)$.

Which height h minimizes the function

$$f(h) = h + \frac{1}{h^2} \ ?$$

Use the second derivative test to check that you have a minimum.

Problem 9) Global extrema (10 points)

An investment problem leads to the profit function

$$f(x) = x - 2x^2 + x^3 \ ,$$

where $x \in [0, 2]$. Find the local and global maxima and minima of f on this interval and use the second derivative test.