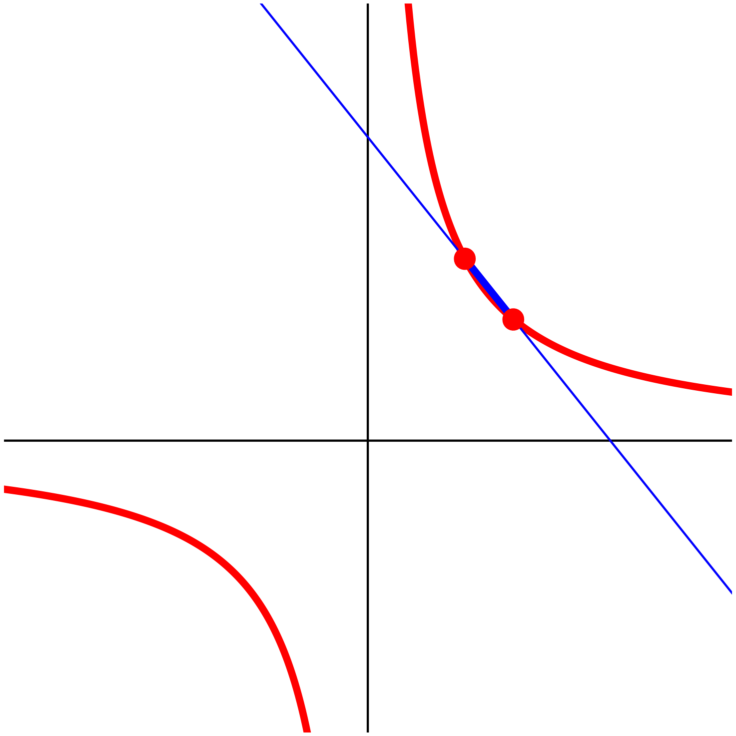
Lecture 7: Worksheet

Rate of change

We compute the derivative of $f(x) = 1/x$ by taking limits.

a) Simplify $\frac{1}{x+h} - \frac{1}{x}$.
b) Now take the limit $\frac{1}{h}[\frac{1}{x+h} - \frac{1}{x}]$ when $h \to 0$.
c) Is there any point where $f'(x) > 0$?



Derivatives

Differentiation rules

$$\frac{d}{dx}x^n = nx^{n-1}$$
$$e^{ax} = ae^{ax}$$
$$\frac{d}{dx}\cos(ax) = -a\sin(ax)$$
$$\frac{d}{dx}\sin(ax) = a\cos(ax)$$

1 Find the derivatives of the function $f(x) = \sin(3x) + x^5$

2 Find the derivative of $f(x) = \cos(7x) - 8x^4$.

3 Find the derivative of $f(x) = e^{5x} + \cos(2x)$.

Two trigonometric identities

During lecture, we need the identities

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

and

$$\sin(x+y) = \cos(x)\sin(y) + \sin(x)\sin(y)\ .$$

You might know these identities from pre-calculus.

We do not work with complex numbers in this course but the verification of these identities is so elegant with the **Euler** formula

$$e^{ix} = \cos(x) + i\sin(x)$$

that it is a crime to prove the identities differently. Just compare the real and imaginary components of

$$e^{i(x+y)} = \cos(x+y) + i\sin(x+y)$$

with the real and imaginary parts of

$$\begin{aligned} e^{ix}e^{iy} &= (\cos(x) + i\sin(x))(\cos(y) + i\sin(y)) \\ &= \cos(x)\cos(y) - \sin(x)\sin(y) + i(\cos(x)\sin(y) + \sin(x)\cos(y))\ . \end{aligned}$$