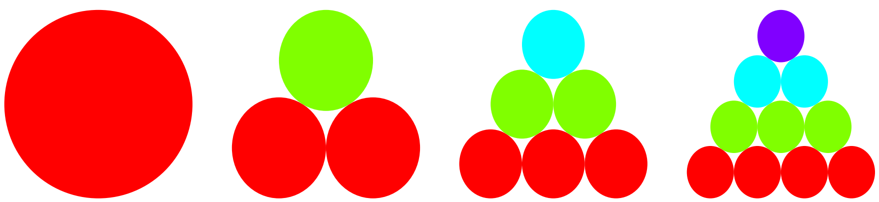
Lecture 1: Worksheet

In this first lecture, we want to see that the **essence** of calculus is already in **basic arithmetic**.

Triangular numbers

We stack disks onto each other building n layers and count the number of discs. The number sequence we get are called **triangular numbers**.

1	3	6	10	15	21	36	45	...



n=1 n=2 n=3 n=4

This sequence defines a **function** on the natural numbers. For example, $f(4) = 10$.

1 Verify that

$$f(n) = \frac{n(n-1)}{2}$$

gives the above numbers. Check this by algebraically evaluating

$$Df(n) = f(n+1) - f(n) .$$

Carl-Friedrich Gauss, 1777-1855

Tetrahedral numbers

We stack now spheres onto each other building n layers and count the number of spheres. The number sequence we get are called **tetrahedral numbers**.

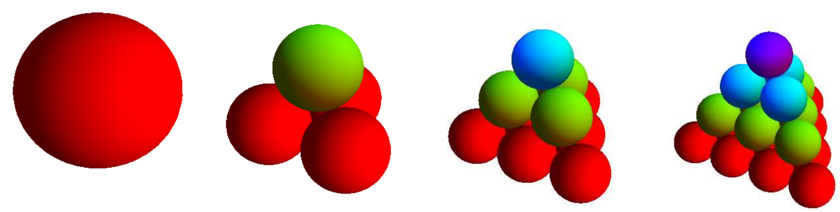
1	4	10	20	35	56	84	120	...

Also this sequence defines a **function**. For example, $g(3) = 10$.

2 Verify that

$$g(n) = \frac{n(n-1)(n-2)}{6}$$

satisfies $g(n+1) - g(n) = n(n-1)/2$.



n=1 n=2 n=3 n=4

You have just verified the formula

$$\frac{d}{dx}[x]^n = n[x]^{n-1}$$

in the case $n = 1, 2, 3$, where $Df(n) = f(n+1) = f(n)$ is the difference and $[x]^n = x(x-1)(x-2)\dots(x-n+1)$ is the "quantum power". In the homework, you push this a bit further.