Lecture 2: Worksheet

In this lecture, we want to learn what a function is and get acquainted with the most important examples.

Trigonometric functions

The cosine and sine functions can be defined geometrically by the coordinates $(\cos(x), \sin(x))$ of a point on the unit circle. The tangent function is defined as $\tan(x) = \sin(x)/\cos(x)$).

$\cos(x) =$ adjacent side/hypothenuse
$\sin(x) =$ opposite side/hypothenuse
$\tan(x) =$ opposite side/adjadcent side

Pythagoras theorem gives us the important identity

$$\cos^2(x) + \sin^2(x) = 1$$

Define also $\cot(x) = 1/\tan(x)$. Less important but sometimes used are $\sec(x) = 1/\cos(x), \csc(x) = 1/\sin(x)$.

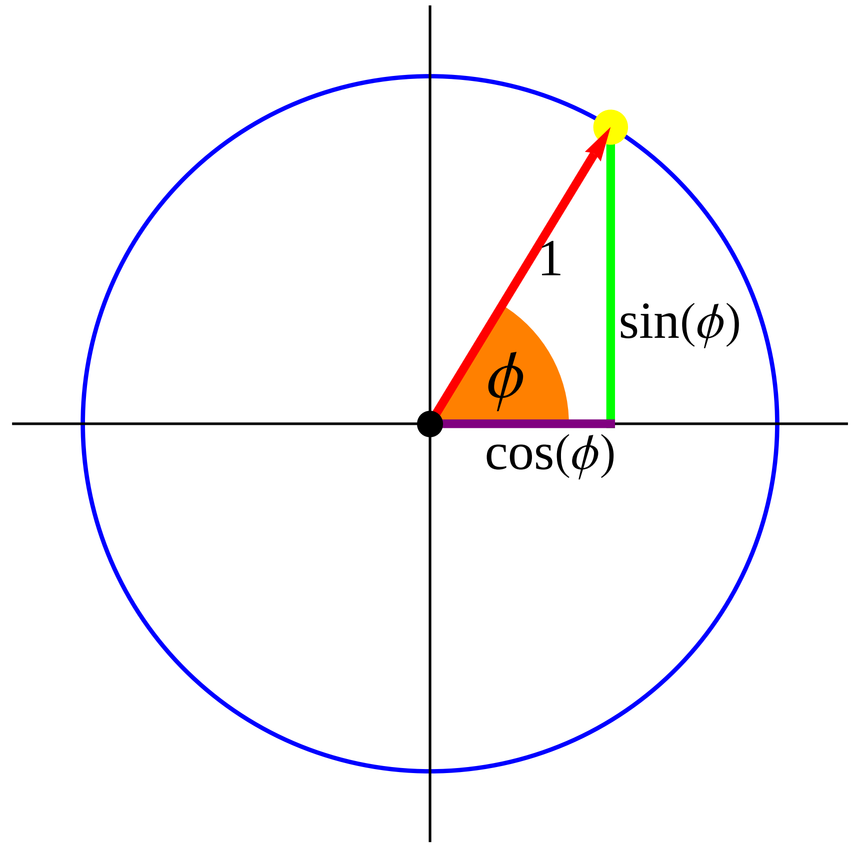
1 Find $\cos(\pi/3)$, $\sin(\pi/3)$.

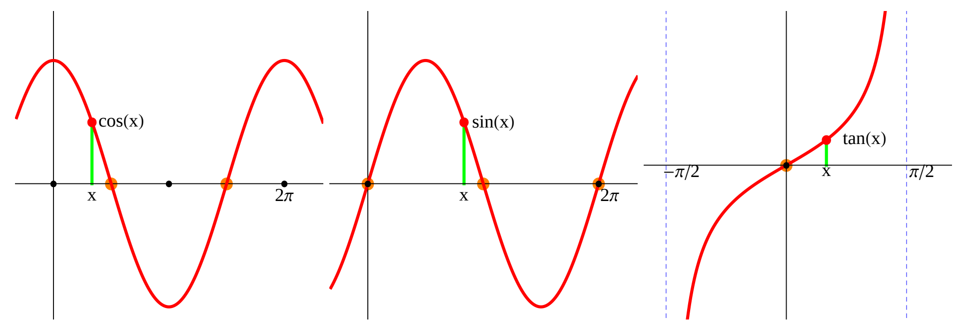
2 Where does cos and sin have roots, places, where the function is zero?

3 Find $\tan(3\pi/2)$ and $\cot(3\pi/2)$.

4 Find $\cos(3\pi/2)$ and $\sin(3\pi/2)$.

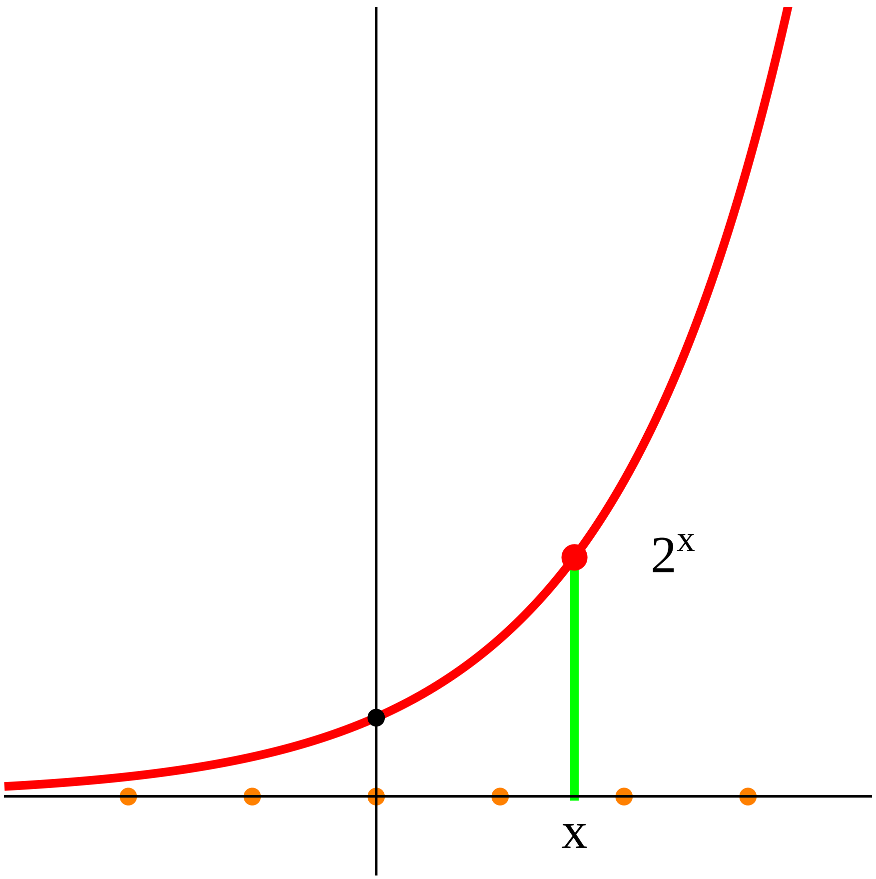
5 Find $\tan(\pi/4)$ and $\cot(\pi/4)$.





The exponential function

The function 2^x is first of all defined for all integers like $2^{10} = 1024$. By taking roots, we can define it for rational numbers like $2^{3/2} = 8^{1/2} = \sqrt{8} = 2.828...$. Since the function 2^x is monotonone on the set of rationals, we can fill the gaps and define the function 2^x for any x. By taking square roots again and again, we see $2^{1/2}, 2^{1/4}, 2^{1/8}, ...$ we approach $2^0 = 1$.



There is nothing special about 2 and we can take any positive base a and define the exponential a^x. It satisfies $a^0 = 1$ and the remarkable rule:

$$a^{x+y} = a^x \cdot a^y$$

It is spectacular because it provides a link between addition and multiplication.

We will especially consider the exponential $\boxed{\exp_h(x) = (1+h)^{x/h}}$, where h is a positive parameter. This is a supercool exponential because it satisfies $\exp_h(x+h) = (1+h)\exp_h(x)$ so that

$$[\exp_h(x+h) - \exp_h(x)]/h = \exp_h(x) .$$

Hold on to that. We will look at this later again. In modern language, we would say that "the quantum derivative of the quantum exponential is the function itself for any Planck constant h".

For $h = 1$, we have the function 2^x we have started with. In the limit $h \to 0$, we get the important exponential function $\exp(x)$ which we also call e^x. For $x = 1$, we get the **Euler number** $e = e^1 = 2.71828...$.

1 What is 2^{-5}?

2 Find $2^{1/2}$.

3 Find $27^{1/3}$.

4 Why is $A = 2^{3/4}$ smaller than $B = 2^{4/5}$? Take the 20th power of both numbers.

5 Assume $h = 2$ find $\exp_h(4)$.