Math 1A: Introduction to functions and calculus Oliver Knill, Spring 2024

2/28/2024: First Hourly Practice A

"By signing, I affirm my awareness of the standards of the Harvard College Honor Code."

Your Name:

Please write neatly. Use the same page for the answer if possible.

1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		100

Problem 1) TF questions (10 points) No justifications are needed.

1) [T] [F] $\lim_{x\to 0} \frac{\sin^2(5x)}{(3x)^2} = \frac{5^2}{3^2}$.

2) [T] [F] $\ln(|7x|)$ has a root at $x = 0$.

3) [T] [F] $\cos(\pi/3) = 1/2$.

4) [T] [F] $f(x) = |5 + 4\sin(x)|$ is differentiable everywhere.

5) [T] [F] $f(x) = \frac{(x^2-2x+1)^3}{(1-x)^2}$ has a limit at $x = 1$.

6) [T] [F] The function $f(x) = |7x|$ has a critical point at $x = 0$.

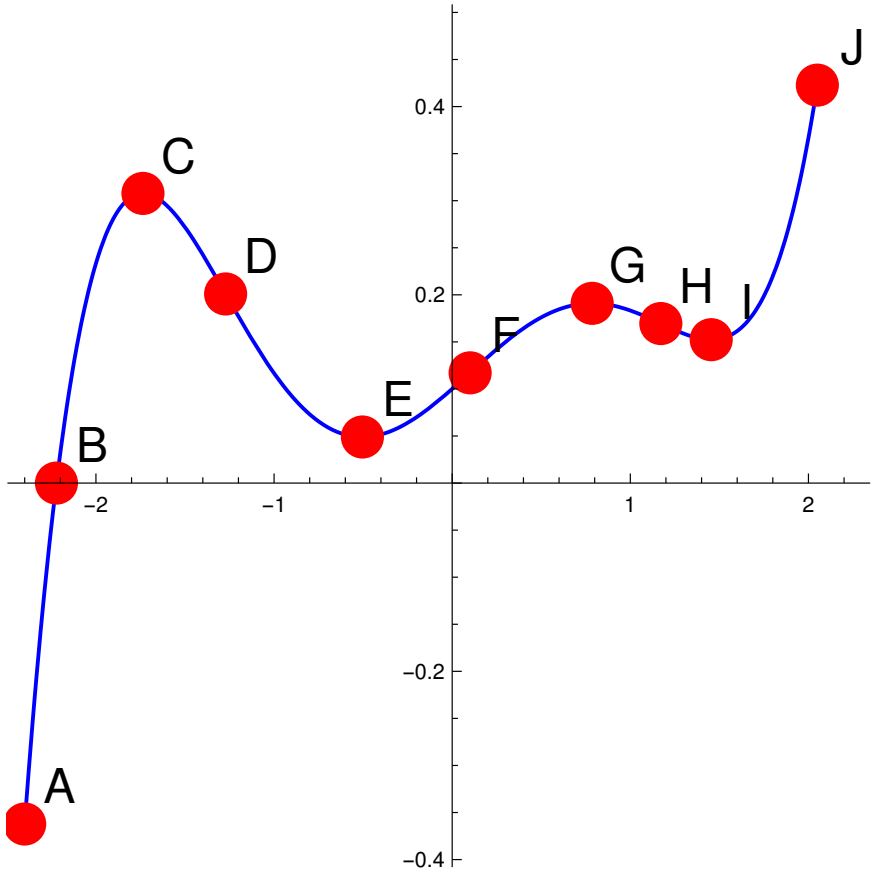
7) [T] [F] The function $f(x) = \cos(\frac{1}{7x})$ with the understanding $f(0) = 0$ is continuous everywhere.

8) [T] [F] The function $(\cos(x) - 1)/x^2$ with the understanding $f(0) = -1/2$ is continuous everywhere.

9) [T] [F] $\frac{d}{dx}\ln(7 - x) = \frac{-1}{7-x}$.

10) [T] [F] The derivative of f/g is $\frac{f'g'-fg}{g^2}$.

Problem 2) Analysis (10 points) No justifications are needed.



a) (2 points) List the points A-J which are roots of f.

b) (2 points) List the points A-J are inflection points.

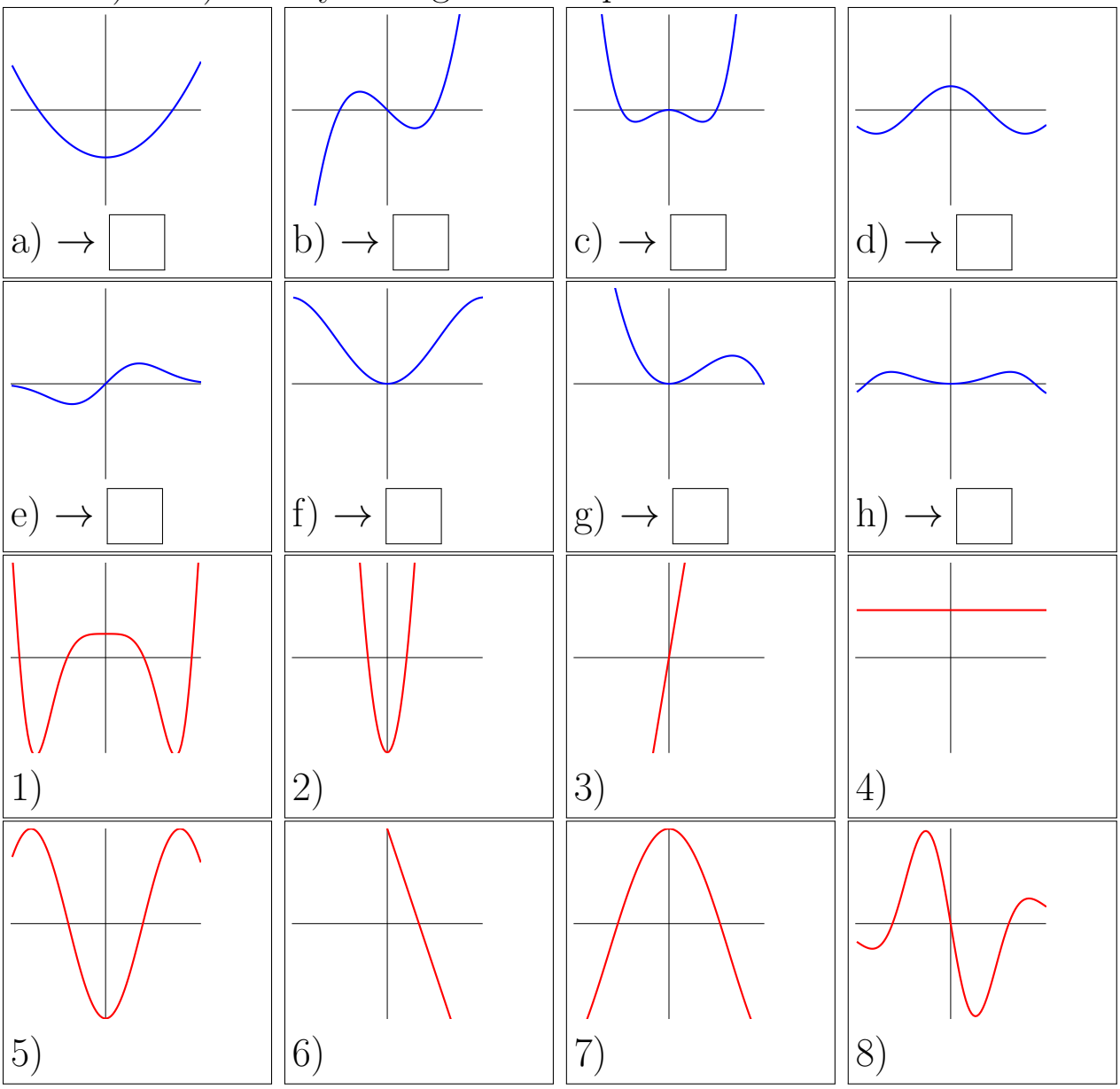
c) (2 points) List the points A-J that are local maxima.

d) (2 points) List the points A-J that are local minima.

e) (2 points) List the points A-J that are global maxima.

Problem 3) Graphing (10 points) No justifications are needed.

Match the functions f a) to h) with the second derivatives functions 1) to 8). Every wrong box is a point off.



Problem 4) Continuity (10 points)

Which of the following functions are continuous on $[-1, 1]$? As usual we extend the domain of definition to points, where a continuation is possible. In each case make the decision "continuous" or "not continuous" and also point to the x-value or values which need special attention.

a) (2 points) $f(x) = \frac{x^6-1}{x^2-1}$

b) (2 points) $f(x) = \frac{\sin(\sin(x))}{\sin(\sin(\sin(x)))}$.

c) (2 points) $f(x) = \frac{\sin^2(x)}{2+\sin(x^2)}$

d) (2 points) $f(x) = \ln|x|e^x$

e) (2 points) $f(x) = \frac{\sin(\tan(x))}{\sin(x)}$

(*) As usual, we extend continuity to functions for which a continuation is possible to initially not defined points, like $f(x) = (x^2 - 1)/(x - 1)$, which is considered continuous everywhere because we can fill in a function value for $x = 1$ which makes it continuous.

Problem 5) Derivatives (10 points)

Find the derivatives of the following functions. In each case, indicate which differentiation rule you use.

a) (2 points) $f(x) = \frac{1}{1+e^x}$.

b) (2 points) $f(x) = \cos(x)\sin(x)$.

c) (2 points) $f(x) = \frac{1+x^3}{1+x^2}$.

d) (2 points) $f(x) = \arctan(7x) + \sin(3x)$.

e) (2 points) $f(x) = \ln(3x) + \ln(5x))$.

Problem 6) Limits (10 points)

Find the limits $\lim_{x\to 0} f(x)$ for each of the following functions:

a) (2 points) $f(x) = \frac{e^{7x}-1}{e^{4x}-1}$.

b) (2 points) $f(x) = \frac{x-1}{x+1}$.

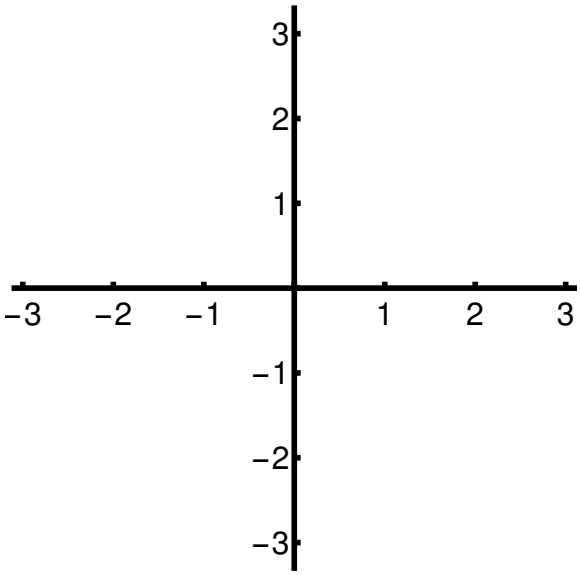
c) (2 points) $f(x) = \frac{\arctan(x)}{\sin(x)}$.

d) (2 points) $f(x) = \frac{\ln(x^3)}{\ln(x^2)}$.

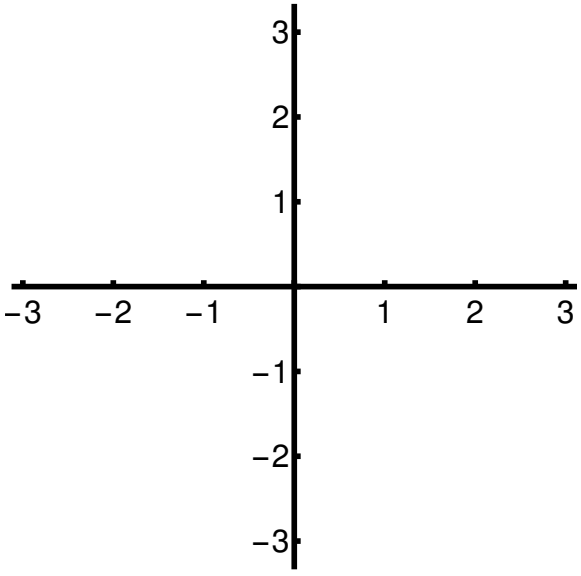
e) (2 points) $f(x) = \frac{\sin(3x)\sin(5x)}{\sin(7x)\sin(2x)}$.

Problem 7) Functions (10 points) no explanations needed

a) (5 points) Draw the graph of the natural log function $f(x) = \log|x| = \ln|x|$. Make sure you get the roots and asymptotes correct.



b) (5 points) Draw the graph of the arctan function $f(x) = \arctan(x)$. Make sure you get the roots and asymptotes correct.



Problem 8) Extrema (10 points)

a) (3 points) Find all the critical points of the function

$$f(x) = x^3 - 3x + 1 \ .$$

b) (3 points) Use the second derivative test to classify the critical points of f.

c) (2 points) On the interval $[-3, 3]$, where it the global maximum, and where is the global minimum?

d) (2 points) Which theorem assures that there is a global maximum and a global minimum on $[-3, 3]$?

Problem 9) Algebra (10 points)

a)	$(e^x)^y$	
b)	e^{x+y}	
c)	$\ln(xy)$	
d)	$\frac{\tan(x)}{\sin(x)}$	
e)	$\frac{x^9}{x^3}$	

Choose from the following expressions.

- $1/\cos(x)$
- $\cos(x)/\sin^2(x)$.
- $e^x e^y$
- $\ln(x+y)$
- $e^x + e^y$
- $e^{(xy)}$
- x^6
- $\ln(x) - \ln(y)$
- $e^{(x^y)}$
- $\log(x) + \log(y)$
- x^3
- $\cos(x)$

Problem 10) Linearization (10 points, 5 points each)

a) (5 points) Use linearization to estimate $\ln(e^2 + 0.1)$.

b) (5 points) Use linearization to estimate $\sin(\frac{\pi}{2} - 0.1)$.