

# INTRODUCTION TO CALCULUS

MATH 1A

UNIT 27: WORKSHEET

## The sigmoid function

**Problem 1:** In some texts, you see the sigmoid function written as  $\sigma(x) = 1/(1 + e^{-x})$ . Verify that this agrees with  $\sigma(x) = e^x/(1 + e^x)$ .

## The fundamental theorem

$$\int_0^x f'(t) dt = f(x) - f(0), \quad \frac{d}{dx} \int_0^x f(t) dt = f(x).$$

**Problem 2:** Write down both expressions in the case  $f(x) = \cos(x)$ .

## The discrete fundamental theorem

Write  $Sf(x) = f(0) + f(1) + \cdots + f(x-1)$ . and  $Df(x) = f(x+1) - f(x)$ .

**Problem 3:** Verify that  $DSf(x) = f(x)$ . This is the analog of  $\frac{d}{dx} \int f(t) dt = f(x)$ .

**Problem 4:** Verify that  $SDf(x) = f(x) - f(0)$ . This is the analog of  $\int_0^x f'(t) dt = f(x) - f(0)$ .

Single Variable Calculus

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