**INTRODUCTION TO CALCULUS**

MATH 1A

UNIT 26: WORKSHEET

# Some True-False Problems

1) | T | **F** | The fundamental theorem of calculus assures that $\int_a^b f'(x)\ dx = f(a) - f(b)$.

**Solution:**
The order is reversed on the right hand side.

2) | **T** | F | If $\int_0^x f(t)\ dt$ is monotonically increasing in $x$ for $0 \leq x \leq 1$, then $f(x) \geq 0$ on $0 \leq x \leq 1$.

**Solution:**
The derivative of the integral is $f(x)$ and because the integral is monotonically increasing, this derivative is positive or zero.

3) | T | **F** | For any continuous function $f$, the integral $\int_a^b f(x)\ dx$ is the area under a curve and therefore always positive or zero.

**Solution:**
It can be a signed area and negative. The statement is false

4) | **T** | F | If $f_c(x)$ has a minimum $x_c$ which is present for $c < 0$ and disappears for $c > 0$, then $c = 0$ is a catastrophe.

**Solution:**
This is a definition.

# A Catastrophe problem

Problem) Catastrophes (10 points)

Consider the family of functions $f(x) = x^3 + cx$ on the real line.
a) (5 points) Find all critical points of $f$, depending on $c$.
b) (2 points) Using the second derivative test, determine which are minima and which are maxima.
b) (3 points) For which value of $c$ does a catastrophe occur?

**Solution:**
a) The critical points are $3x^2 + c = 0$ which means $x = \sqrt{-c/3}$. There are no critical points for $c > 0$ and two different critical points for $c < 0$.
b) The second derivative is $6x$ which is negative for $x < 0$ and positive for $x > 0$. The point $\sqrt{-c/3}$ is a local minimum for $c < 0$. The point $-\sqrt{-c/3}$ is a local maximum for $c < 0$.
c) The catastrophe appears at the parameter $c = 0$ because a local minimum, present for $c < 0$ disappears for $c \geq 0$.

Problem) Integrals (10 points)

Two of the following 6 integrals can be solved (with the methods we know) two more are harder to guess at this stage. Two can not be solved.
a) $\int \ln(x)\, dx$
b) $\int 1/\ln(x)\, dx$
c) $\int \ln(x)/x\, dx$
d) $\int 1/(\ln(x)x)\, dx$
e) $\int x/\ln(x)\, dx$
f) $\int x\ln(x)\, dx$

**Solution:**
a) $x\ln(x) - x + C$. (will be solve later)
b) Can not be solved
c) $\ln(x)^2/2 + C$
d) $\ln(\ln(x)) + C$
e) Can not be solved
f) $-x^2/4 + x^2\ln(x)/2 + C$ (will be solved later)

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