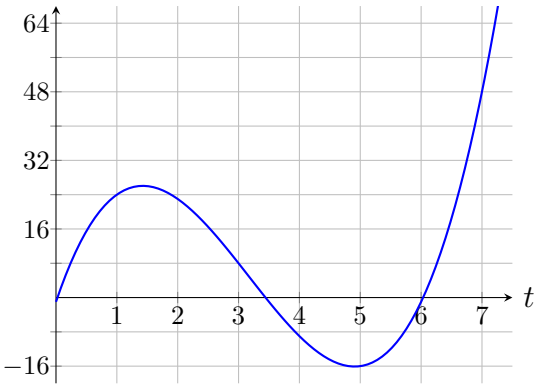
INTRODUCTION TO CALCULUS

MATH 1A

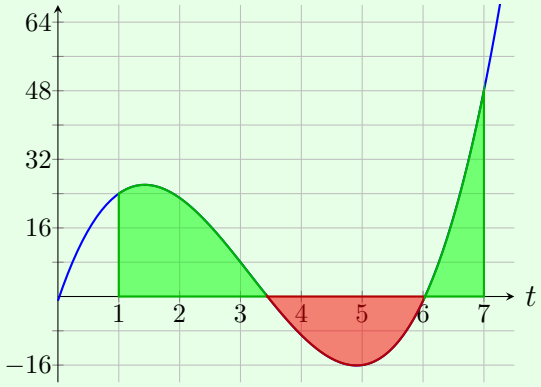
UNIT 23: WORKSHEET

Riemann Sums

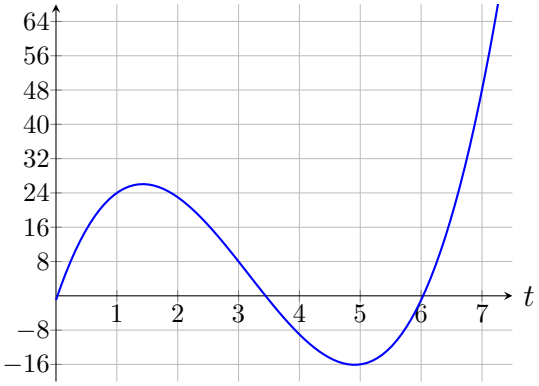
Problem 1: The number of people in the math 4th floor common room changes over time, as people get in and out. Suppose $f(t)$ gives the net change of people at time t. Are there more folks in the department at 7 PM than at 12 PM? To do so, look at the areas you see in the picture.



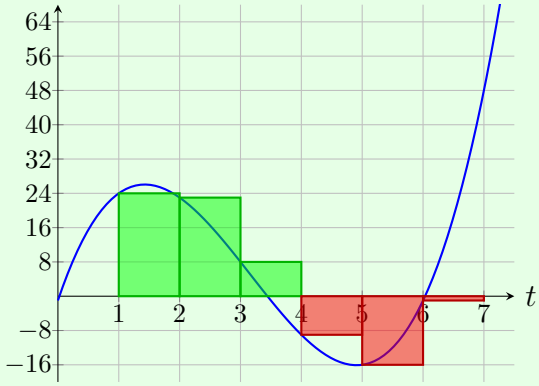
Solution:
We can visualize the exact net change in population between $t = 1$ and $t = 7$ as the signed area between the curve and the x-axis. That's the area of the green regions below minus the area of the red region:



Problem 2: Now do a left Riemann sum with rectangles of width $\Delta x = 1$ and give a rough estimate how many folks there are in the department at 7 PM.
P.S. You sum up 5 rectangles from $x_0 = a = 1$, $x_1 = 2$, to $x_5 = 6$ because $x_k = a + k(b-a)/6$ and $a = 1$ and $b = 7$.



Solution:



Lets look at the notation. We have $\Delta x = 1$ and $[a, b] = [1, 7]$. This determines the points $x_0 = a = 1$ and $x_1 = a + \Delta x = 2$ and $x_2 = a + 2\Delta x = 3$ and $x_3 = a + 3\Delta x = 4$ and $x_4 = a + 4\Delta x = 5$ $x_5 = a + 5\Delta x = 6$ and $x_6 = b = a + 6\Delta x = 7$. Our sum consists of 6 parts as we have split up the interval into 6 smaller intervals of length 1. There are first three positive contributions, then two negative contributions and then one contribution 0 as $f(6) = 0$. We have about $S_6 f = 24 + 22 + 8 - 8 - 16 + 0 \sim 30$.