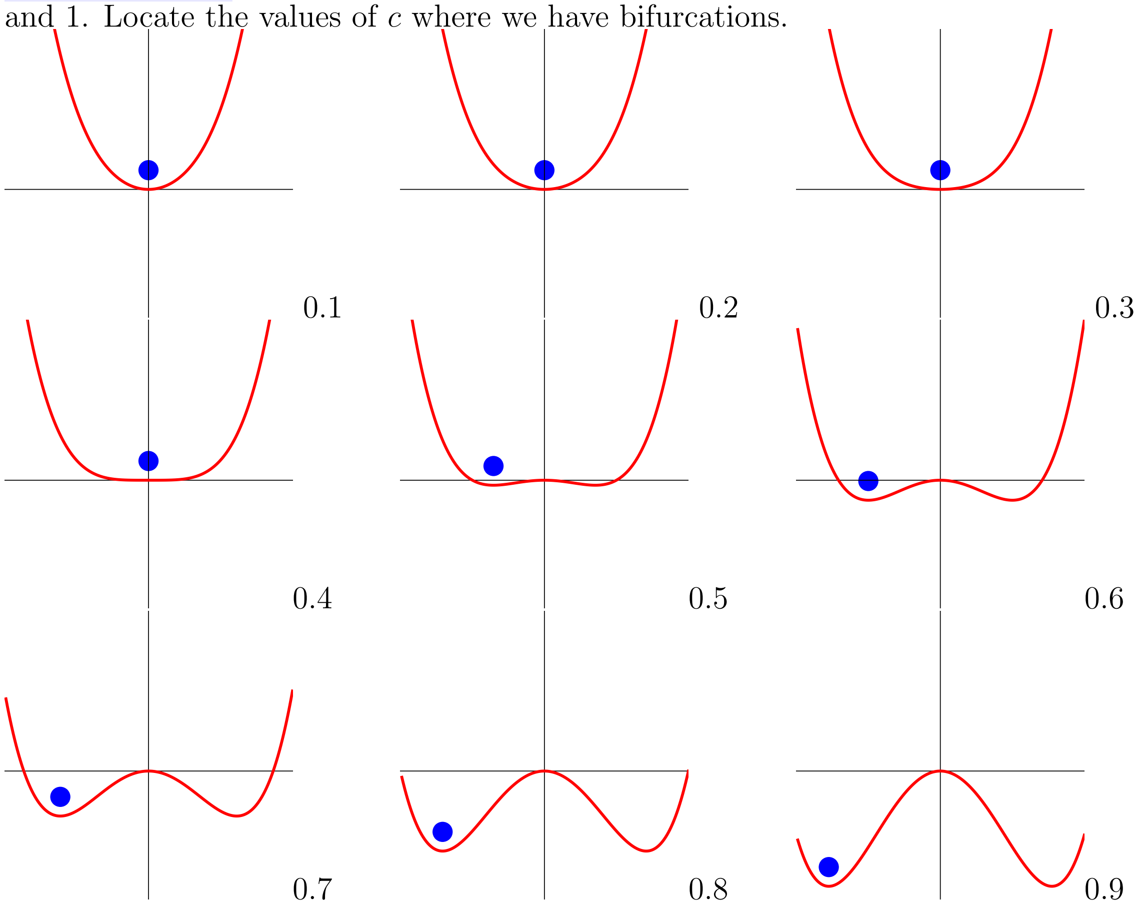
INTRODUCTION TO CALCULUS

MATH 1A

Unit 22: Worksheet

Catastrophes

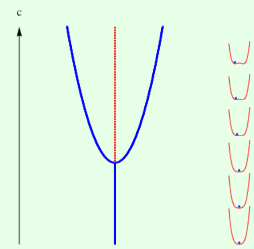
Problem 1: We see here graphs of the function $f(x) = x^4 - cx^2$ for c between 0 and 1. Locate the values of c where we have bifurcations.



Solution:
This is the example we looked at in class. For $c = 0$ we have only one critical point at $x = 0$. Take the derivative $f'(x) = 4x^3 - 2cx = 0$. This is zero if $x = 0$ or if $x = \sqrt{c/2}$. We see that there are 3 critical points for $c > 0$ and one critical point at $c \leq 0$. The bifurcation point is $c = 0$.

Problem 2: Draw the bifurcation diagram in this case. The vertical axes is the c axes.

Solution:
The diagram looks like a fork. It is a pitchfork bifurcation. In this worksheet we only look from c=0 to c=1. The case c=0 is the value of the bifurcation.

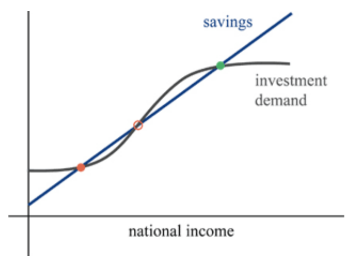


Dangerous intersections

From Strogatz (NYT 2012): (http://opinionator.blogs.nytimes.com/2012/10/08/dangerous-intersection):

"In economics, a similar mechanism underlies the jumps predicted by a model of the business cycle. The idea goes back to **Nicholas Kaldor***, a disciple of Keynes, in the 1940s, and was recast in the framework of catastrophe theory in 1979 by Hal Varian, currently the chief economist at Google. The assumptions behind the model - and their limitations - are explained in detail here and in Varian's elegant paper, so let me focus on the model's tangential intersections and what they suggest about the economy.*

In these diagrams of the economy, the horizontal axis shows the level of economic activity, as reflected by the national income. Values on the far right mean a booming economy, while values on the left mean a sluggish economy like the one we've been in for the past few years. The model says that the economy will be in equilibrium when the supply of funds available for investment matches the demand for such funds. On the graph, equilibrium occurs where the "savings line" crosses the "investment curve."



Depending on how the line and curve are situated, one, two or three intersections can occur. The upper intersection (the green dot) represents a strong economy with high levels of national income. The lower equilibrium (solid red dot) depicts an economy stuck in the doldrums. The middle equilibrium (open red circle) turns out to be unstable and acts like a watershed; when economic conditions are near it, they drift away toward one of the other two equilibriums. Now comes the crucial idea. The amount of investment doesn't depend only on national income but also on how much investment has already been accumulated. At some point enough is enough. During the housing boom, for example, increases in income fueled the demand for housing investment. But as the stock of housing rose, the demand for investment dropped. In the model this drags the S-shaped investment curve down. It's like the straw being added to the camel's back. As the next animation shows, that little straw - that gradual lowering of the demand for investment - can suddenly tip a strong economy into recession".
(See http://vimeo.com/49423671)."

So much about Strogatz. You can try this out with Mathematica.

```
Manipulate[Plot[{x,Cos[2x]+x/2+2.4-a},{x,0,6}],{a,0,2}]
```

Oliver Knill, knill@math.harvard.edu, Math 1a, Spring, 2024