INTRODUCTION TO CALCULUS

MATH 1A

Unit 18: Worksheet

17.1. **Problem 1:** A couple of weeks ago was **Valentine day**. The heart function $(x^2+y^2-1)^3 - x^2y^3 = 0$ relates x with y, but we can not write the curve as a graph of a function $y = y(x)$. Extracting y or x is difficult. We still can find the derivative y' knowing $x = 1, y = 1$.

Solution:
Using the chain rule, we can take the derivative

$$3(x^2+y^2-1)(2x+2yy') - 2xy^3 - x^2 3y^2 y' = 0$$

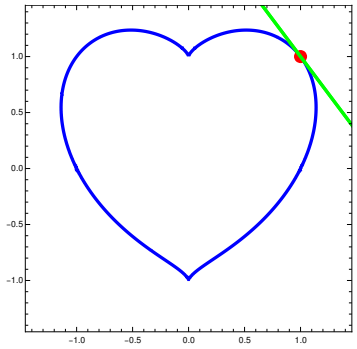
We can now solve solve for y'

$$y'(x) = -\frac{3(x^2+y^2-1)2x - 2xy^3}{3(x^2+y^2-1)2y - 3x^2y^2} .$$

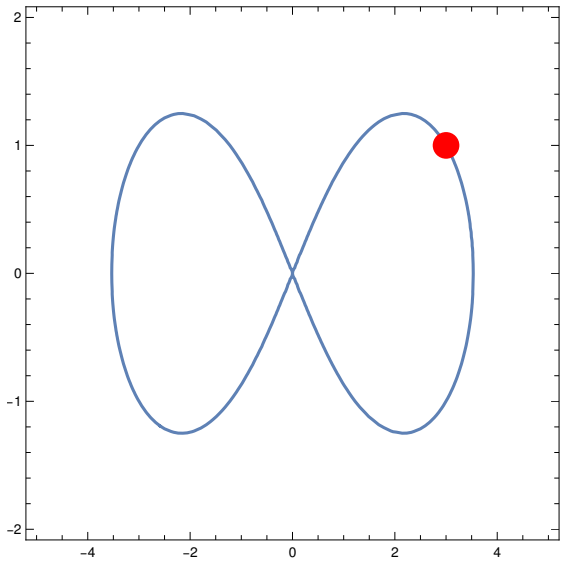
Filling in $x = 1, y = 1$ gives $y' = -4/3$. We have computed the slope of g without knowing g. Magic! It comes even better as we have seen in class: we can already after taking the derivative plug in x=1 and y=1. And get

$$3(1+1-1)(2+2y') - 2 - 3y' = 0$$

from which we can solve for $y' = -4/3$ faster without all the gymnastics.



Problem 2: The lemniscate curve is given by $2(x^2+y^2)^2 = 25(x^2-y^2)$. Find the slope of the tangent at $(3, 1)$. This is the derivative of y' at $x = 3$.



Solution:
Take the derivative $4(x^2+y^2)(2x+2yy') = 50x - 50yy'$. Now plug in $x=3, y=1$ to get $\boxed{y' = -9/13}$.

Oliver Knill, knill@math.harvard.edu, Math 1a, Spring, 2024