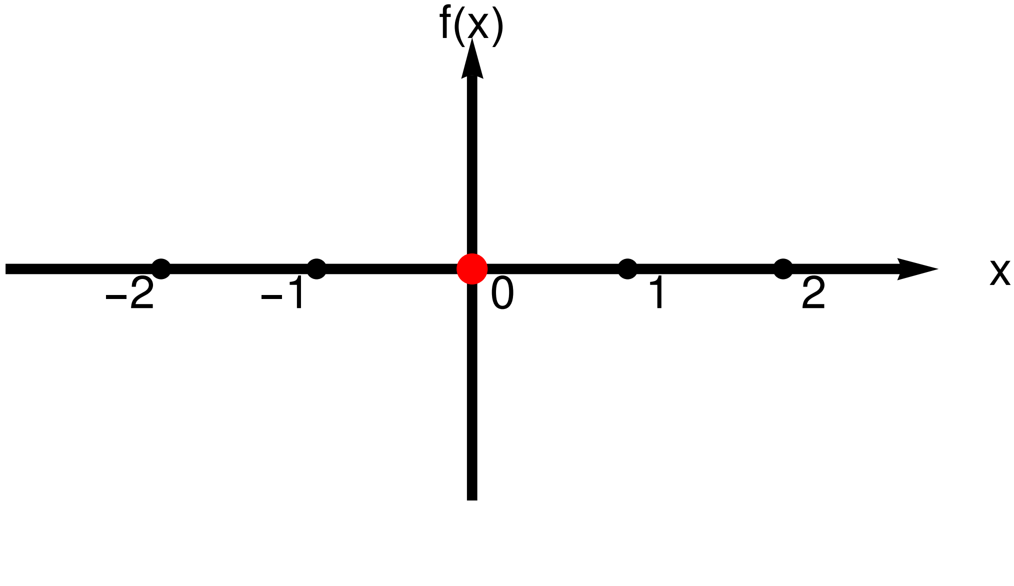
INTRODUCTION TO CALCULUS

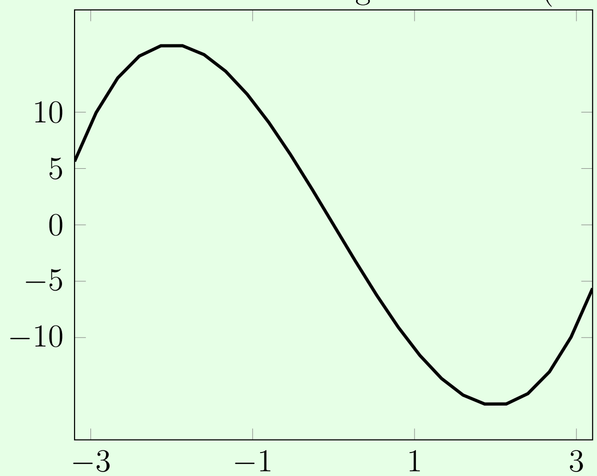
MATH 1A

Unit 12: Worksheet

Problem 1: Let $f(x) = x^3 - 12x$.
a) Find all critical points of f.
b) Use the first derivative test to find the maxima and minima.
c) Use the second derivative test to find maxima and minima.
d) Now use this information to draw the graph of f.

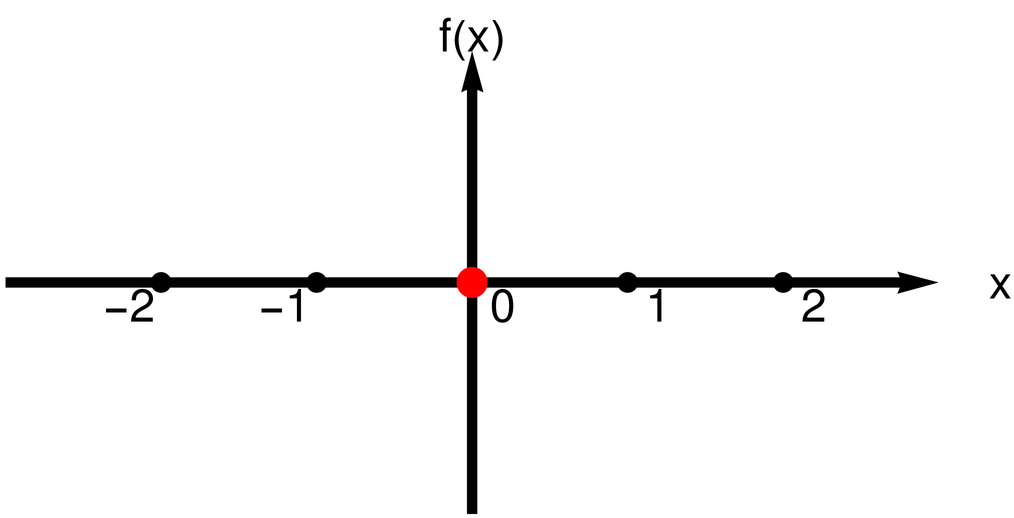


Solution:
a) $x = 2$ and $x = -2$. b) The first derivative test gives a local minimum at -2 and a local maximum at 2 c) The second derivative is $3x$ which is negative at -2 (maximum) and

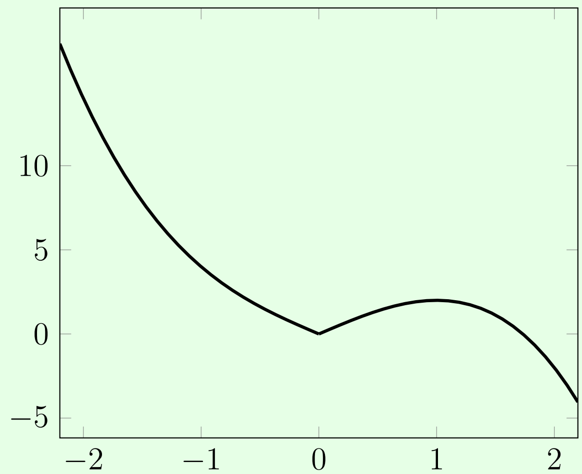


positive at $x = 2$ (minimum) d)

Problem 2: Find the maxima and minima of the function $f(x) = 3|x| - x^3$ following the same path as in the previous problem. To get started, first look at what happens for $x > 0$ and then for $x < 0$.

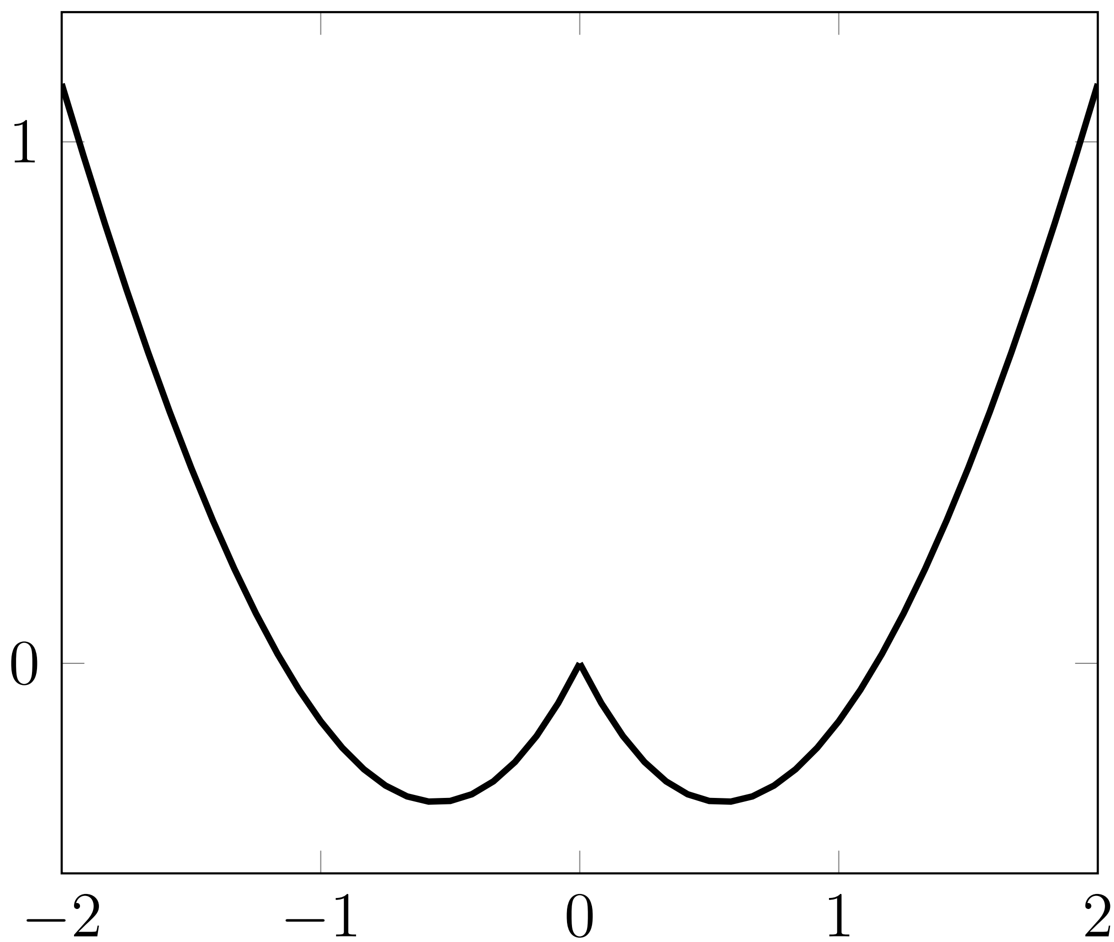


Solution:
For $x > 0$ the function is $3x - x^3$ which can be differentiated. The derivative $3 - 3x^2$ is zero at $x = 1$. For $x < 0$ the function is $-3x - x^3$. The derivative is $-3 - x^2$ and has no root. The only critical point is 1. The second derivative is $-6x = -6$ at this point so that by the second derivative test, the point is a local max. There is also the point $x = 0$. We can differentiate the function there. We have to deal with this point separately. The first derivative test tells us that it is a minimum. The reason is that for $x < 0$, the slope of the function is $-3 - 3x^2$ which is negative and that for $x > 0$, the slope of the function is $3 - 3x^2$ which is positive on $(0, 1)$. We can also see from the first derivative test that 1 is a local max because $f'(x) < 0$ for $x > 1$. The function is monotonically decreasing on $(-\infty, 0)$, monotonically increasing on $(0, 1)$ and monotonically decreasing for $(1, \infty)$.



Problem 3:
You see the graph of a continuous function f. It is called the **Big W** function.

a) identity the intervals where f is increasing
b) identify the intervals where f is decreasing.
c) identify the intervals where f is concave up.
d) identify the intervals where f is concave down.
e) Which points are critical points of f?
f) Which points are local maxima?
g) Which points are local minima?



Solution:
a) Increasing from -0.7 to 0, from 0.7 to 2
b) Decreasing from -2 to -0.7 and from 0 to 0.7
c) From -2 to 0 and from 0 to 2
d) nowhere
e) -0.7, 0.7
f) 0 but the derivative does not exist here.
g) -0.7, 0.7

Oliver Knill, knill@math.harvard.edu, Math 1a, Spring, 2024