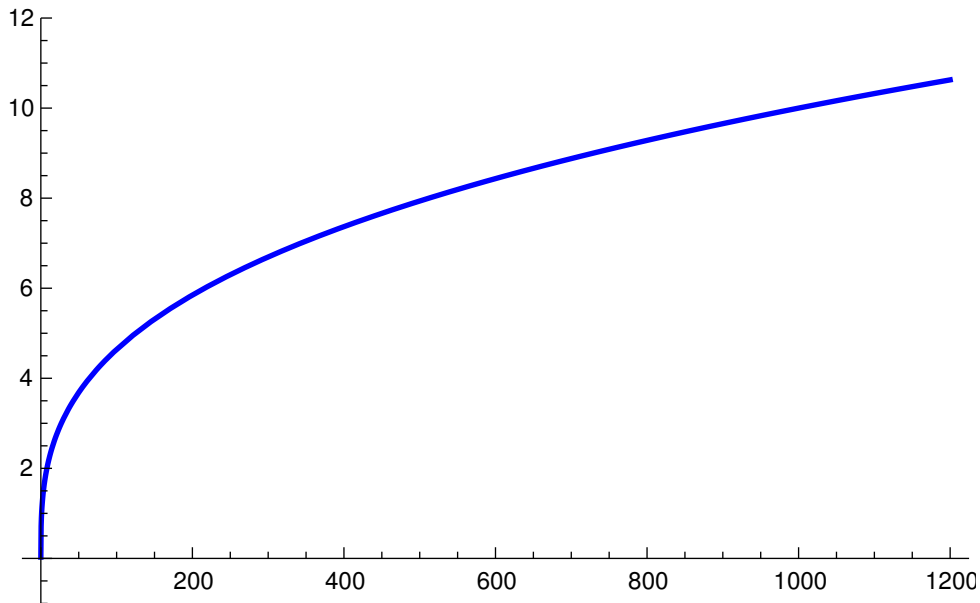


# INTRODUCTION TO CALCULUS

MATH 1A

## UNIT 11: WORKSHEET

**Problem 1:** The following graph shows the cube root function  $f(x) = x^{1/3}$ . Locate the point  $a = 512$ , its value  $f(a)$  and draw the linearization of  $f$  at  $a$ . How close to you estimate the error  $f(600) - L(600)$  just by looking at the graphs? Can you compute the difference explicitly?



**Solution:**

When looking at the picture,  $f(600) - L(600)$  is small, smaller than  $1/10$ . The linearization is  $L(x) = 8 + (x - 512)/(3 * 64)$  which gives 8.46 The actual number is 8.434 which is maybe 0.03 off.

**Problem 2:** Estimate  $\sqrt{150}$  using linear approximation at  $a = 144$ ?

**Solution:**

$$12 + 6/24 = 12.25$$

**Problem 3:** Estimate  $\ln(2)$  using linear approximation at  $a = 1$ .

**Solution:**

We have  $L(x) = \ln(1) + (1/1)(x - 1)$  which gives the value 1.  $\ln(2)$  is smaller 0.65. here the linear approximation is not that good. But 2 is very far off from 1.

**Problem 4:** Estimate  $\sin(0.1)$  using linear approximation.

**Solution:**

$L(x) = 0 + 1(x - 0) = x$ . The estimate is 1.

**Problem 5:** Estimate  $\arctan(1.1)$  using linear approximation at  $a = 1.0$ .

**Solution:**

We have to compute  $\arctan(1) = \pi/4$  and  $\arctan'(1) = 1/2$ . The linearization is  $\pi/4 + 1/2(x - 1)$ . We get  $\pi/4 + 0.1/2$

**Problem 6:** Estimate  $\tan(0.3)$  using linear approximation at  $a = 0$ .

**Solution:**

$\tan(0) = 0, \tan'(0) = 1$ . The linear approximation function is  $L(x) = x$ . The estimate is 0.3.