INTRODUCTION TO CALCULUS

MATH 1A

Unit 10: Worksheet

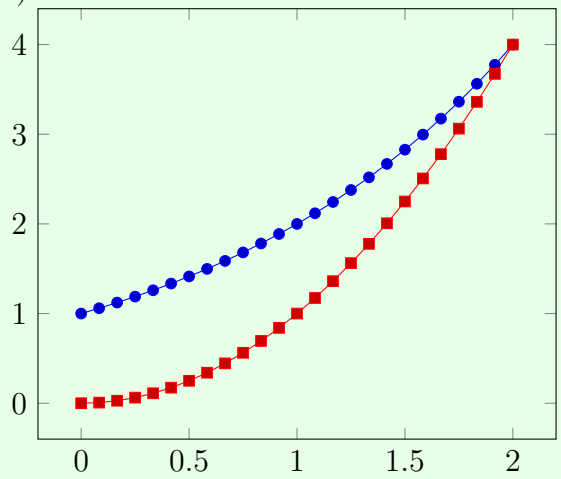
Problem 1: In this lecture we are interested in **infinity**. This means especially that we are interested in **large numbers**. Lets see who comes up with the largest number made of 5 digits or symbols. Your number has to be finite!

Solution:
Good suggested solutions were 99^{99} (454 digits), 9^{999} (2195 digits), 9999! (82000 digits), (9!)! (4.2 Million digits) The best submission was 9^{99} (with 850 Million digits). With the Knuth notation (see homework 5), one can contribute $9 \uparrow\uparrow\uparrow 9 = 9 \uparrow\uparrow 9 \uparrow\uparrow 9 \uparrow\uparrow 9 \uparrow\uparrow 9 \uparrow\uparrow 9 \uparrow\uparrow 9 \uparrow\uparrow 9 \uparrow\uparrow 9 \uparrow\uparrow 9$. Note that $9 \uparrow\uparrow 9 = 9^{9^{9^{9^{9^{9^{9^{9^{9}}}}}}}}$ has already an insane amount of digits (around $9^{9^{9^{9^{9^{9^{9^{9}}}}}}}$ digits). We believe that this is the largest number one can get like that.

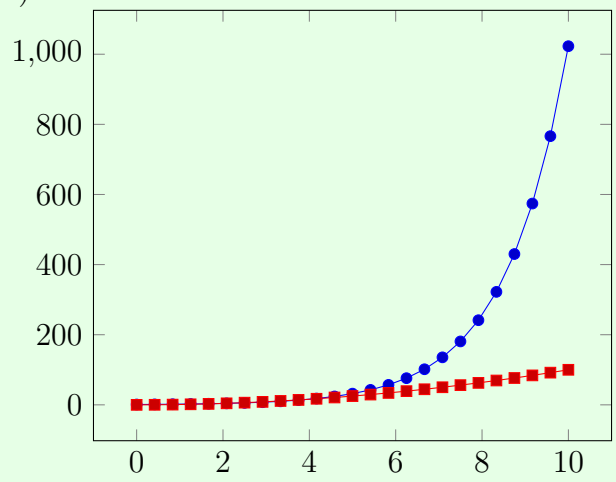
Problem 2: a) Draw the graph of $f(x) = x^2$ and $g(x) = 2^x$ on the interval $[0, 2]$. b) Now draw these graphs on the interval $[0, 10]$. There is some space to draw on the back of this worksheet.

Solution:

a)



b)



Problem 3: What is $\lim_{x\to\infty} \frac{x^2}{2^x}$. Compute this using the l'Hospital rule.

Solution:

0. After bringing twice to the hospital.

Problem 4: Compute the limit:

$$\lim_{x\to\infty} \frac{\sin(3x)+x}{x^2+x}$$

Solution:
0

Problem 5: This had been done last time already but it actually should have belonged to this lecture Evaluate the following limit:

$$\lim_{x\to 1}(x-1)/\log(x-1)$$

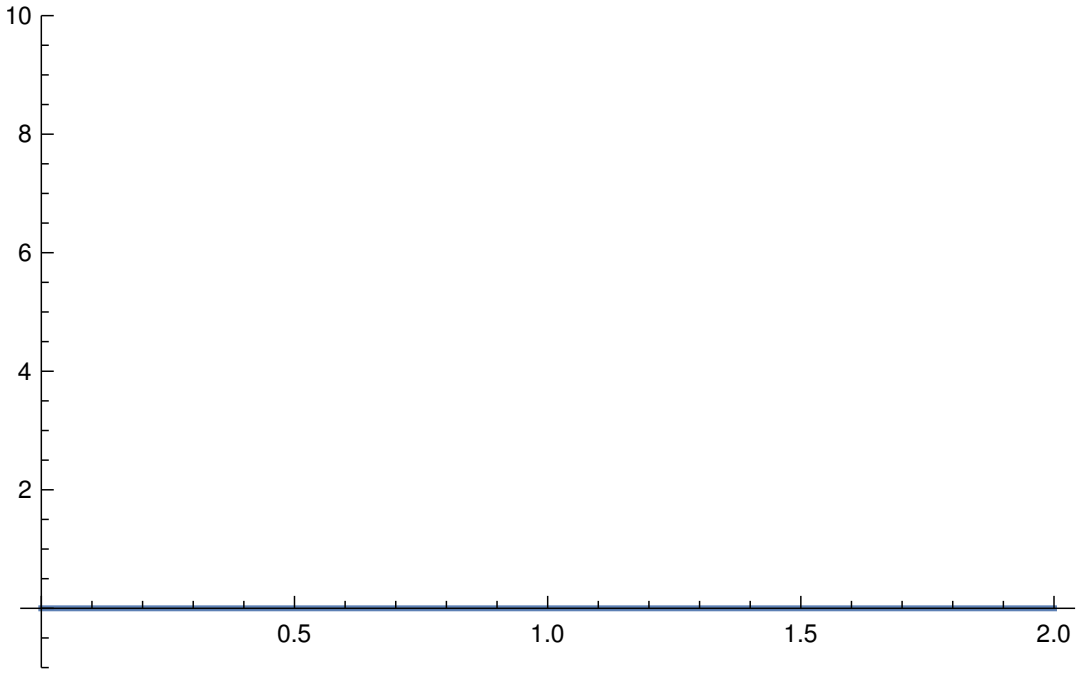
Solution:
L'Hospital gives 0. We should check however, whether this is really a case for Hospital. The example is not appropriate because it is not of the form 0/0. It is of the form 0/infinity which is zero.

Problem 6: Find the limit:

$$\lim_{x\to\infty}\log(2x+3)/\log(5x+1)$$

Solution:
Hospital and then again applying Hospital gives 1.

Graph for Problem 2a)



Graph for Problem 2b)