INTRODUCTION TO CALCULUS

MATH 1A

Unit 6: Worksheet

Problem 1: Take the definition of the derivative to find the derivative of the function $f(x) = 2x$.

Problem 2: Take the derivative of $f(x) = x^4$ by taking limits. We have to simplify

$$\frac{f(x+h) - f(x)}{h}.$$

Start with expanding $(x+h)^4$.

Problem 3: Take the definition of the derivative to look at

$$\frac{f(x+h) - f(x)}{h}$$

in the case $f(x) = |x|$. What happens at $x = 0$?

Problem 4: We start here with the task to find the derivative of $f(x) = \ln(x)$ by taking limits. Simplify $[\ln(x+h) - \ln(x)]$ as much as possible.

We will continue to work on this next week.

Problem 5: In the famous **bottle calibration problem**, we fill a circular bottle or glass with constant amount of fluid. Plot the height of the fluid in the bottle at time t. Assume the radius of the bottle is $f(z)$ at height z. Can you find a formula for the height $g(t)$ of the water? This is not so easy. But we can find the rate of change $g'(t)$. Assume for example that f is constant, then the rate of change is constant and the height of the water increases linearly like $g(t) = t$. If the bottle gets wider, then the height of the water increases slower. There is definitely a relation between the rate of change of g and f. Before we look at this more closely, lets try to match the following cases of bottles with the graphs of the functions g qualitatively. In each of the bottles, we call g the height of the water level at time t, when filling the bottle with a constant stream of water. Can you match each bottle with the right height function.