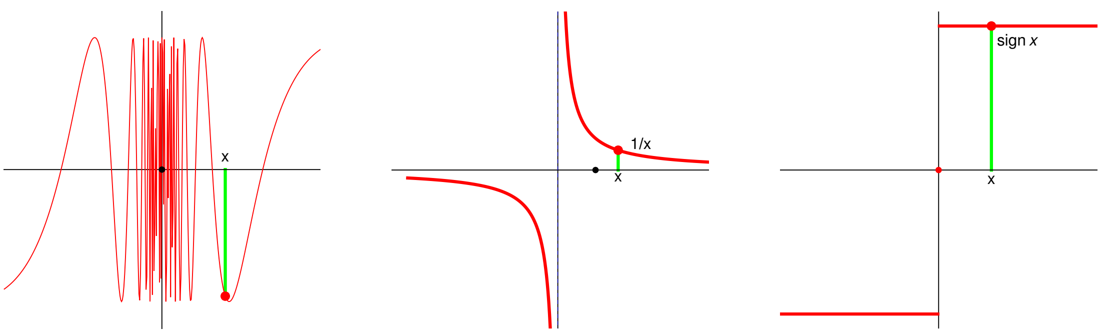
INTRODUCTION TO CALCULUS

MATH 1A

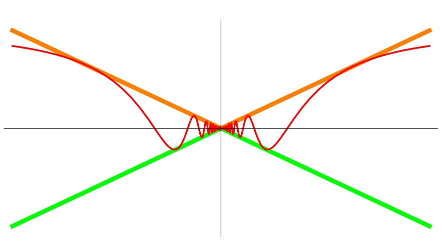
UNIT 5: WORKSHEET

There are a few mechanisms for discontinuity . A function can **jump for good**, **badly rush to infinity** or have an **ugly oscillation**. All three cases come from a division by zero somewhere.

Nice Guys	**Good,Bad,Ugly Guys**
$x^2 + 4x + 6$	$1/x$ at 0
$\sin(x), \cos(x)$	$\tan(x)$ at $\pi/2$
$\exp(x)$	$\log\lvert x\rvert$ at 0
$\text{sinc}(x) = \frac{\sin(x)}{x}$	$\frac{1}{\cos(x)}$ at $\pi/2$



Squeeze theorem



Lets look at the **tamed devil** $f(x) = x\sin(\frac{1}{x})$. A bad oscillation is tamed by the **squeeze theorem**: We have $-|x| \leq x\sin(\frac{1}{x}) \leq |x|$ and both $g(x) = |x|$ and $h(x) = -|x|$ are continuous at 0 and have the same value at 0. Because f is sandwiched between two continuous functions which come together at the point, the function must be continuous and give that common value.

Which functions are continuous?

If you can assign a value at a point where it is not defined so that it becomes overall continuous, we just consider this continuous. We consider $(x^4-1)/(x-1)$ to be continuous for example because it is equivalent for $x \neq 1$to x^3+x^2+x+1 which is continuous. We consider $\sin(x)/x$ to be continuous because we can fill in a value 1 at $x=0$ to make it continuous overall. We saw that $x\sin(x)$ is continuous everywhere because of the squeeze theorem. Which of the following functions are continuous?

Problem 1: $f(x) = x^2 + x^2\sin(1/x^2)$

Problem 2: $f(x) = \sqrt{|x|}$

Problem 3: $f(x) = |x^3|/x^3$

Problem 4: $f(x) = |x|^2/x$

Problem 5: $f(x) = \frac{1}{\sqrt{|x|}}$

Problem 6: $\frac{1}{\log|1/x|}$

Problem 7: $\log(\log|x|)$

Problem 8: $1/(1+|x|)$

Problem 9: $1/(1-|x|)$

Problem 10: $x^2/\sin(x)$

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