# INTRODUCTION TO CALCULUS

MATH 1A

## Unit 4: Worksheet

**Problem 1:** Does the function $\frac{\cos(x)}{x}$ have a limit at $x \to 0$? If yes, what is it? If not, why does the limit not exist?

**Solution:**
No. As $\cos(x) \to 1$ (a finite value) and $1/x \to \infty$, there is no chance of a limit, when we take the product $\cos(x)/x$.

**Problem 2:** Does the function $\frac{\sin(x)}{e^x}$ have a limit at $x \to 0$? If yes, what is it? If not, why does the limit not exist?

**Solution:**
Yes. The limit is zero. Since $\sin(x) \to 0$ and $e^x \to 1$, we do not have a problem.

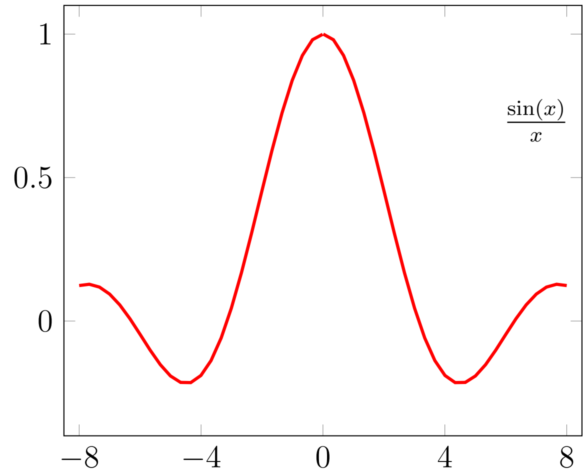
**Problem 3):** A prototype function for studying limits is the sinc function

$$f(x) = \frac{\sin(x)}{x} .$$

At which points can you be sure that the function has a limit? We will investigate the limiting behavior in class theoretically.

**Solution:**
At all points except $x = 0$. At $x = 0$ we have seen that the limit is 1.

**Problem 4):** First some experiment. Lets look at the graph of the function




If you look at the graph, does it appear that the function has a left or/and right limit everywhere?

**Solution:**
Yes, the graph looks smooth at $x = 0$. Whatever software we use, whether Desmos or Mathematica. There is appears to be no problem at $x = 0$.

**Problem 5:** Now that you know the answer to $\lim_{x \to 0} \sin(x)/x$, find the $\lim_{x \to 0} \frac{\sin(x^2)}{x^2}$.

**Solution:**
Yes, 1. Just treat $y = x^2$ as a new variable. Since $\lim_{y \to 0} \frac{\sin(y)}{y} \to 1$, we also have $\lim_{x \to 0} \frac{\sin(x^2)}{x^2} \to 1$.

**Problem 6:** Does the function $\frac{\sin(x^2)}{x}$ have a limit for $x \to 0$?

**Solution:**
Yes,0. We can write this as $x\frac{\sin(x^2)}{x^2}$. Since $x \to 0$ and $\frac{\sin(x^2)}{x^2} \to 1$, we have $x\frac{\sin(x^2)}{x^2} \to 0$.

**Problem 7:** Does the function $\frac{\sin(x)}{x^2}$ have a limit for $x \to 0$?

**Solution:**
No,it goes to infinity. We can see that as a product of $\text{sinc}(x)$ and $1/x$. The first function has a limit, the second does not have a limit.

**Problem 8:** Does the function $\frac{x}{\sin(x)}$ have a limit for $x \to 0$?

**Solution:**
Yes,1. Since $\sin(x)/x \to 1$, we also have $x/\sin(x) \to 1$.

**Problem 9:** Does the function $\frac{\sin(x)}{|x|}$ have a limit for $x \to 0$?

**Solution:**
No,there is a jump. We can write this as $\frac{\sin(x)}{|x|} = \sin(x)/x(x/|x|)$. The next problem 10) shows that $x/|x| = \text{sign}(x)$ does not have a limit. So, also the product does not have a limit.

**Problem 10:** Does the function $\frac{x}{|x|}$ have a limit for $x \to 0$?

**Solution:**
No,there is a jump. This function $x/|x|$ is the $\text{sign}(x)$ function. It is 1 if $x > 0$ and $-1$ if $x < -0$. There is no limit therefore.