INTRODUCTION TO CALCULUS

MATH 1A

Unit 1: Worksheet

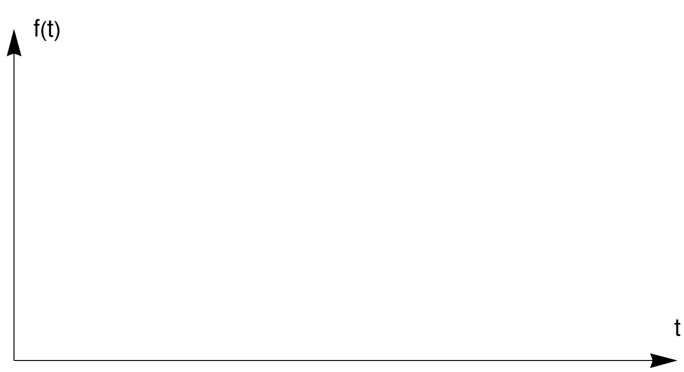
Problem 1):

a) Pick your favorite ball:

- tennis
- golf
- soccer
- baseball
- softball
- lacrosse
- marble
- pinball

Explain to your neighbors why you made your pick.

b) Imagine to throw the ball and display its trajectory $f(x)$ as a function of time x. Draw a possible trajectory.



c) Identify parts of the graph, where the function is increasing, decreasing, where it is concave down and concave up.

d) If you would play with the ball on the moon rather than the earth. How would that change the trajectory? How would the concavity change?

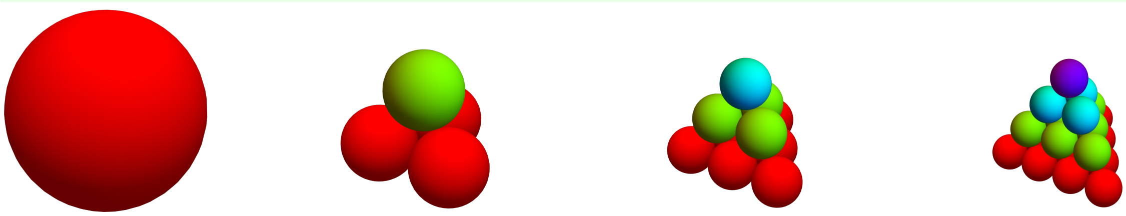
e) Identify the units of $x, f(x)$ the unit for the slope and concavity have?

Solution:
a) Your pick. My favorite ball is a billiard ball.
b) A possible trajectory is already displayed on the handout for this lecture.
c) The ball follows parabolic paths, the function is increasing, reaching a temporary top, then decreases. At the bouncing points, it changes from decreasing to increasing. All the arcs are concave down. At the impact point the concavity is not defined.
d) On the moon, the gravity is 6 times lower, the concavity is 6 times smaller too. e) x could be measured in seconds, $f(x)$ in meters, the velocity in meters per second, the concavity in meters per second squared.

Problem 2): If we stack oranges onto each other, we are led to **tetrahedral numbers**. Can you find the next number?

$$1 \ \ 4 \ \ 10 \ \ 20 \ \ 35 \ \ 56 \ \ 84 \ \ 120 \ \ ...$$

Solution:
The next term is 165.



n=1 n=2 n=3 n=4

Problem 3): How does the following sequence

$$0 \ \ 6 \ \ 24 \ \ 60 \ \ 120 \ \ 210 \ \ 336 \ \ 504 \ \ \dots$$

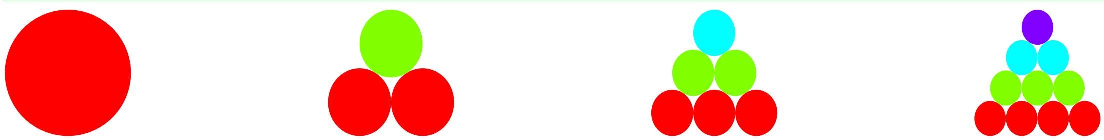
continue? We look at this as a function $f(1) = 0, f(2) = 6, \dots, f(8) = 504$. What is the next term $f(9)$?

Solution:
The next term is 720.

Problem 4): When adding the first integers we get the so called **triangular numbers**. This sequence defines a **function** on the natural numbers. For example, $f(4) = 1 + 2 + 3 + 4 = 10$. Can you guess a formula for $f(x)$?

$$1 \ 3 \ 6 \ 10 \ 15 \ 21 \ 36 \ 45 \ \ldots$$

Solution:
This is what Gauss got. He had to add all the first 100 numbers 1+2+3+....+ 100. His idea was to pair things up. 1+100 + 2+99 + ...+ (50+51) which is 50*101. In general it is (n/2)(n+1).



n=1 n=2 n=3 n=4

Oliver Knill, knill@math.harvard.edu, Math 1a, Spring, 2024