INTRODUCTION TO CALCULUS

MATH 1A

Unit 26: Review

Important results

Chain rule: $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$.

Implicit differentiation: $f(x,y) = c$ allows to compute y' if x, y are given

Extremal value theorem: a continuous function on $[a,b]$ has a max and min.

Intermediate value thm: continuous f on $[a,b]$ with $f(a) * f(b) < 0$ have roots.

Related rates: a rule relating $x(t)$ and $y(t)$ determines $y'(t)$ if $x'(t)$ is given.

Mean value theorem: a differentiable f on $[a,b]$ has x with $f'(x) = \frac{f(b)-f(a)}{b-a}$

Rolle's theorem: a differentiable f on $[a,b]$ with $f(a) = f(b)$ has a critical point.

Newton step: The step $T(x) = x - f(x)/f'(x)$ allows to get closer to a root of f.

Catastrophes: parameter values c, where the number of minima decreases.

Definite integrals: $\int_a^b f(x)\,dx$ is defined as a limit of Riemann sums.

Anti derivative: $F(x) = \int_0^x f'(t)\,dt$ satisfying $F' = f$.

Indefinite integral: given by $F + C$, where C is a constant and $F' = f$.

Fundamental theorem of calculus: $\int_a^b f'(x)\,dx = f(b) - f(a)$.

Fundamental theorem of calculus: $\frac{d}{dx} \int_0^x f(t)\,dt = f(x)$.

Signed area: $\int_a^b f(x)\,dx$ area between graph and x-axes, area below counted negative.

Important integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{1}{x} = \ln(x) + C$$

$$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + C$$

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \frac{1}{\cos^2(x)}\,dx = \tan(x) + C$$

$$\int \frac{-1}{\sin^2(x)}\,dx = \cot(x) + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$$

$$\int \tan(x)\,dx = -\ln(\cos(x)) + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$$

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \text{arcsinh}(x) + C$$

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