

INTRODUCTION TO CALCULUS

MATH 1A

Unit 0: Warm Up: due 1/22/2024

Problem 1.1: Algebra: Please simplify the following expressions

- a) $\frac{1}{\frac{1}{3} - \frac{1}{4}}$
- b) $(1/3)/(4/9)$
- c) $\sqrt{x^6}x/x^{-2}$
- d) $((2^4)^3) - (2^3)^4$
- e) $\ln(e^{10}e^{11})$.

Solution:

- a) 12
- b) $3/4$
- c) x^6
- d) 0
- e) 21

Problem 1.2: Equations: Please solve the following equations for x .

- a) $x^2 + 1 = 2x$
- b) $\sin(x) = 1/2$
- c) $2^x = 1$
- d) $\tan(x) = \sin(x)$
- e) $\sqrt{x} + x = x\sqrt{x}$.

Solution:

- a) $x = 1$.
- b) $\pi/6 = 30^\circ$.
- c) $x = 0$.
- d) $\sin(x) = 0$ or $\cos(x) = 1$ which means $x = k\pi$ with integer k .
- e) $(\sqrt{5} + 3)/2$.

Problem 1.3: Graphing: Graph the following functions

- a) $f(x) = 3x + 2$
- b) $f(x) = 2^x - 5$
- c) $f(x) = x^3 - x$
- d) $f(x) = e^{-x^2}$
- e) $f(x) = \sin(2x) + 1$

Solution:

For this problem you can plot it by hand, once you know what happens. Good things to look for are how the function grows, especially in the long term.

- a) This is a line of slope 3 passing through (0,2)
- b) This is an exponential function which monotonically increases goes asymptotically to -4 for $x \rightarrow -\infty$ and to ∞ for $x \rightarrow \infty$.
- c) This is a cubic function which comes from -infinity, goes to a max, then a min and then goes to infinity.
- d) this is the gaussian (bell curve). It is never zero, never negative and goes to zero on both ends.
- e) this is a wave. It is never negative but has roots when $\sin(2x) = -1$ which is at $3\pi/4 \pm k\pi/2$.

Problem 1.4: Geometry: A triangle has side lengths 3, 4, 5.

- a) What is the largest angle in the triangle?
- b) What is the area of the triangle?
- c) What is $\arcsin(4/5)$?
- d) Solve the equation $\arctan(3/4) = \arcsin(x)$ for x .
- e) What is $\arccos(3/5) + \arcsin(3/5)$?

Solution:

- a) 90°
- b) $3 * 4/2 = 6$
- c) $12/3 + 12/4 + 12/5 = 9.4$
- d) Use the triangle $\arctan(3/4) = y$ is an angle if this is $\arcsin(3/5)$ then $\sin(y) = 3/5 = 0.6$.
- e) The sum of the two smaller angles of the triangle is $\pi/2 = 90$ degrees.

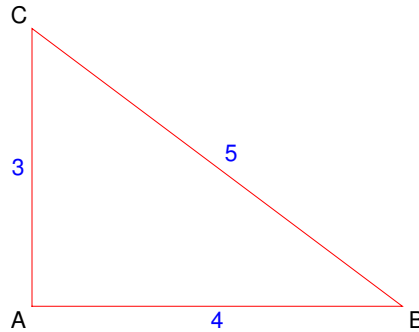


FIGURE 1. The 3-4-5 triangle.

Problem 1.5: Exponentials: Which of the following statements are true?

if $x + y = z$?

- a) $2^x + 2^y = 2^z$
- b) $2^x 2^y = 2^z$
- c) $2^{x+y} = 2^z$
- d) $x^2 + y^2 = z^2$
- e) $\log_2(x) + \log_2(y) = \log_2(z)$

Solution:

- a) False
- b) True
- c) True d) False e) False

Problem 1.6: Modeling: You count 1000 bacteria of a colony today $t = 0$ and 32000 after $t = 5$ days. How many bacteria are there in 9 days?

Solution:

$10002^{kt} = f(t)$. We see that $k = 1$. So, in 9 days there are $1000 \cdot 2^9$ bacteria.

Problem 1.7: Laws: a) Write down 3 laws for exponentiation.
b) Write down 3 laws for logarithms.

Solution:

a) $a^x * b^x = (ab)^x$

$(a^x)^y = a^{xy}$

$a^0 = 1$

b) $\log(xy) = \log(x) + \log(y)$

$\log(a^x) = x \log(a)$

$\log(1) = 0$

Problem 1.8: Definitions: Summarize the definitions for $\sin(x)$, $\cos(x)$, $\tan(x)$ and $\cot(x)$.

Solution:

SOH-CAH-TOA-CAO

Problem 1.9: Triangles: There are 2 important special triangles for which all angles are known explicitly. What are these angles. (You should find 4).

Solution:

45,30,60,90

Problem 1.10: Please check the syllabus if necessary:

- What are the names of your instructors and CA's
- Summarize the homework policies.
- Is Chat GPT allowed in this course?
- When are the midterm dates?
- State the first sentence of the Harvard honor code.

Solution:

- a) Oliver, Eugenia, Jota and Jacobo
- b) Due every class. No late HW policy, least 3 PS scores are deleted.
- c) Not for HW and exams.
- d) Feb 28 and April 3.
- e) "Members of the Harvard College community commit themselves to producing academic work of integrity."

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