35

FINAL REVIEW

MATH 1A, UNIT 35

Oliver Knill,
Harvard University

April 26 2021

PLAN

1. Poll

2. Big picture

3. Review of early parts

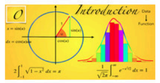
4) Theorems

5. Applications

6. Jam

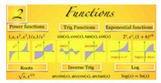
POLL

What unit was the hardest?

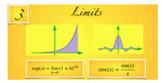


Unit 00

Unit 01

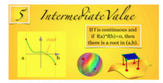


Unit 02



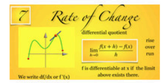
Unit 03

Unit 04



Unit 05

Unit 06



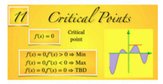
Unit 07


Data 01

Unit 08

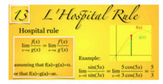
Unit 09

Unit 10

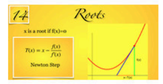


Unit 11

Unit 12

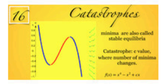


Unit 13

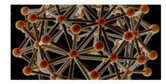


Unit 14

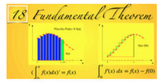
Unit 15



Unit 16

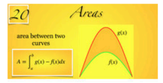

Data 02

Unit 17

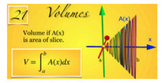


Unit 18

Unit 19

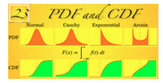


Unit 20



Unit 21

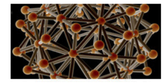
Unit 22



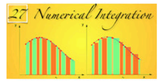
Unit 23

Unit 24

Unit 25


Data 03

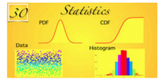
Unit 26



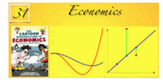
Unit 27

Unit 28

Unit 29

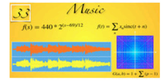


Unit 30

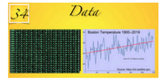


Unit 31

Unit 32



Unit 33



Unit 34


Data 04

PLANNING

Reading Period

Exam

Office hours

1A SONG

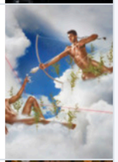
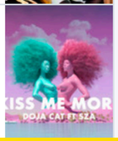
Good bye, math 1a,
we reach the end
of the season.

Good times, I have to say,
we have had all
for a reason.

You are wiser now,
having got fed all this
brain food.

Go in peace now,
use it for working on
doing good!

NOT IN TOP CHART!

1 New	**Rapstar** Polo G	⊙	-	1	1	
2 →	**Montero (Call Me By Your Name)** Lil Nas X		2	1	3	
3 ↓	**Leave The Door Open** Silk Sonic (Bruno Mars & Anderson .Paak)		1	1	6	
4 ↓	**Peaches** Justin Bieber Featuring Daniel Caesar & Giveon	★	3	1	4	
5 ↑	**Save Your Tears** The Weeknd	⊙	6	4	18	
6 ↑	**Levitating** Dua Lipa Featuring DaBaby	⊙	7	5	28	
7 New	**Kiss Me More** Doja Cat Featuring SZA	⊙	-	7	1	








CLASS FOTO!

THANKS!

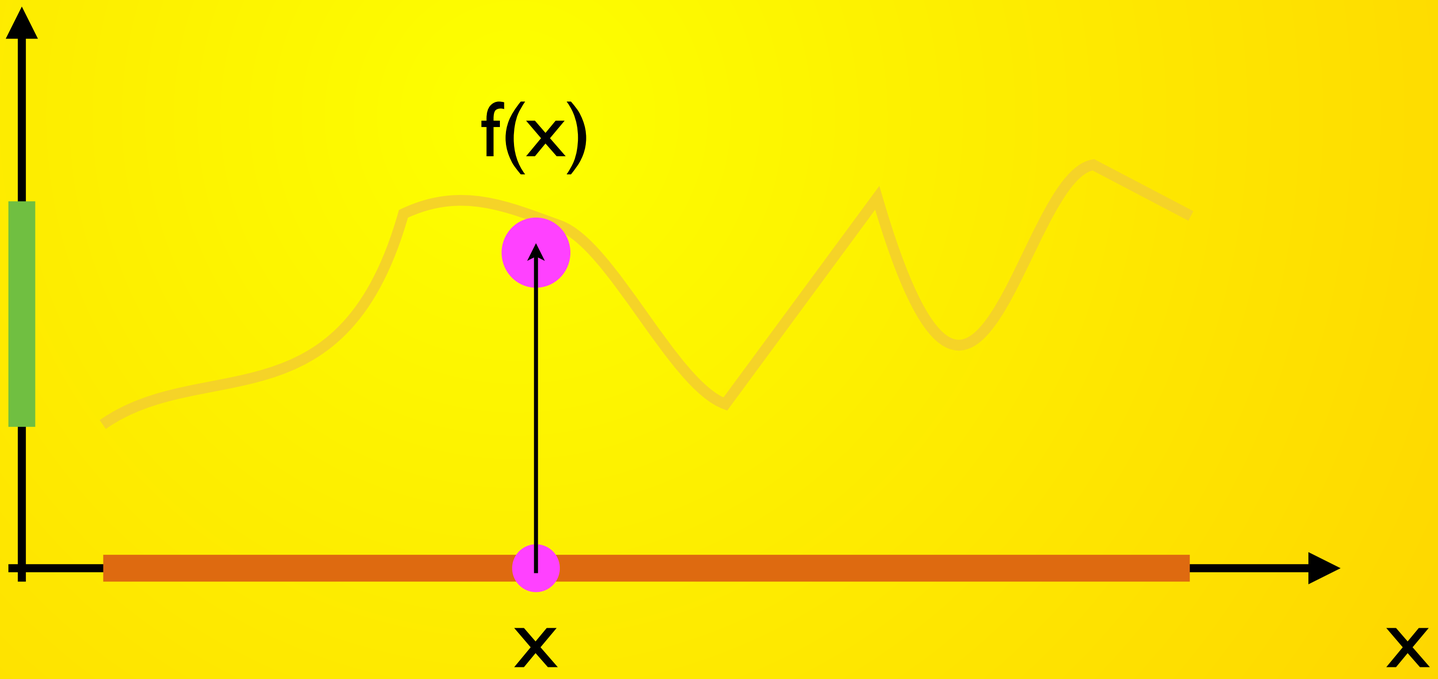
Jackie

Keith

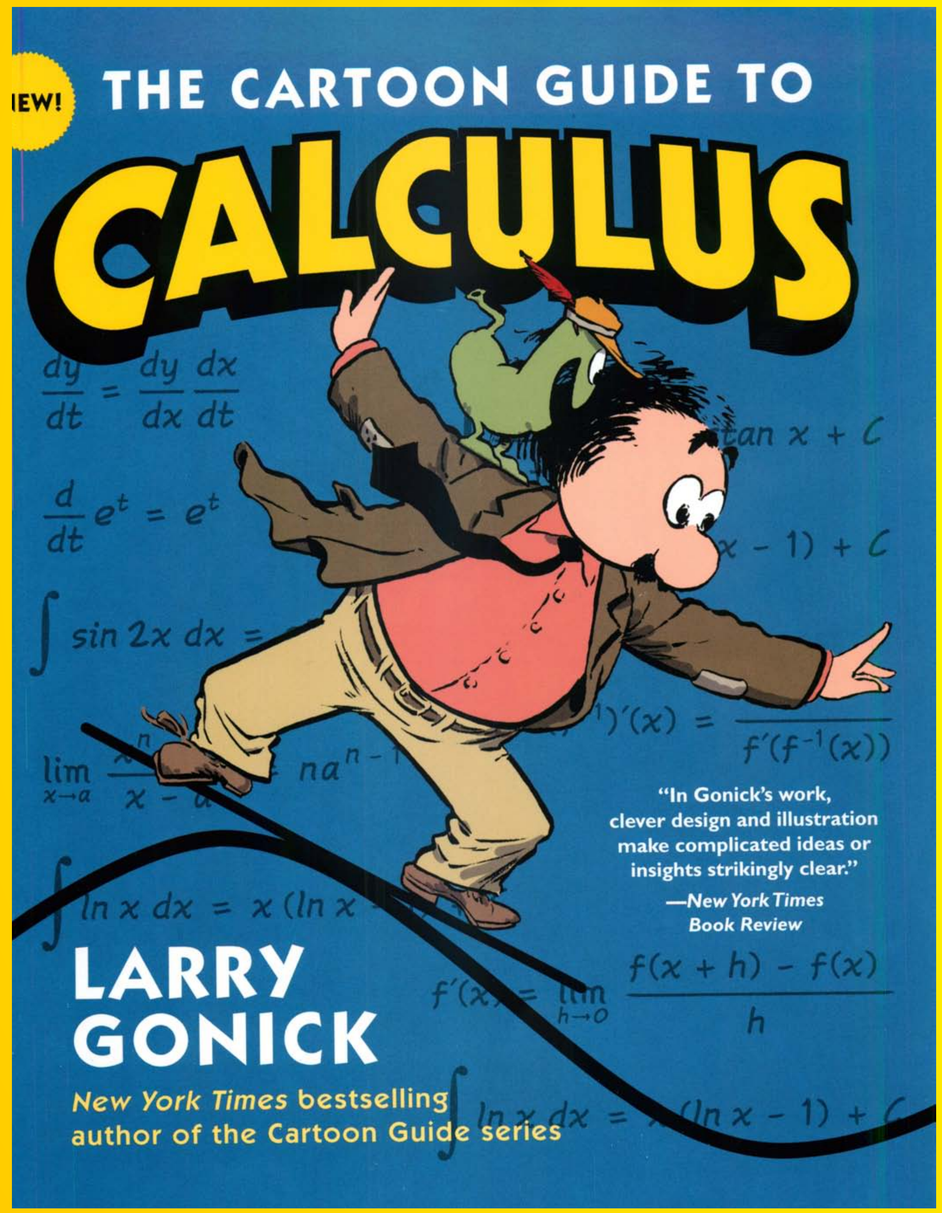
Michaela

FUNCTIONS

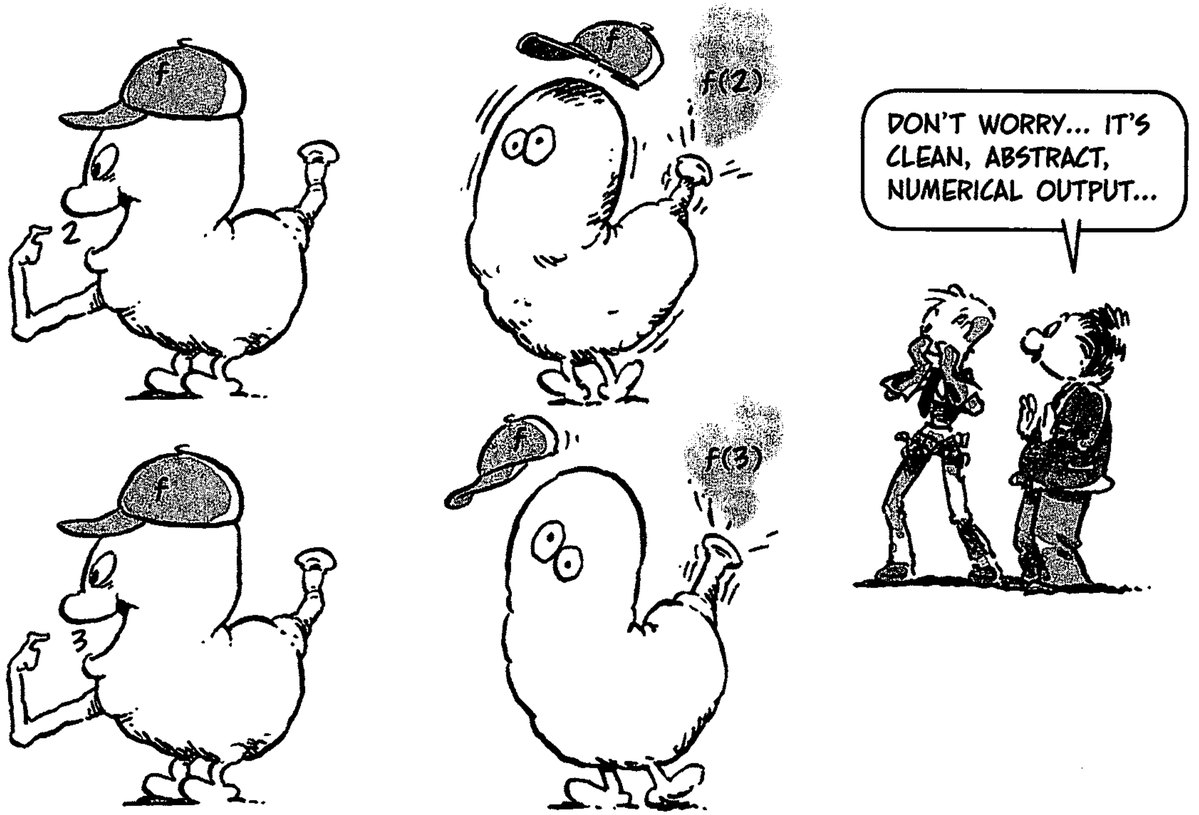
=



INPUT-OUTPUT



A FUNCTION IS A SORT OF **INPUT-OUTPUT DEVICE** OR **NUMBER-PROCESSOR.** A FUNCTION (CALL IT f) EATS AND SPEWS NUMBERS IN A SPECIFIC WAY. FOR EACH NUMBER EATEN (CALL IT x), f OUTPUTS A SINGLE, UNIQUE NUMBER, $f(x)$, PRONOUNCED "EFF OF ECKS." f IS LIKE A RULE THAT TRANSFORMS x INTO $f(x)$. IN GOES x, OUT COMES $f(x)$.



IF YOU DON'T LIKE YOUR OUTPUT FLOATING AROUND IN THE AIR LIKE SWAMP GAS, THEN THINK OF NUMBERS AS LYING ALONG A LINE. IN THAT CASE, YOU CAN IMAGINE A FUNCTION f EATING NUMBERS FROM ONE LINE AND MERELY **POINTING** TO THE CORRESPONDING OUTPUT VALUES ON THE OTHER LINE.

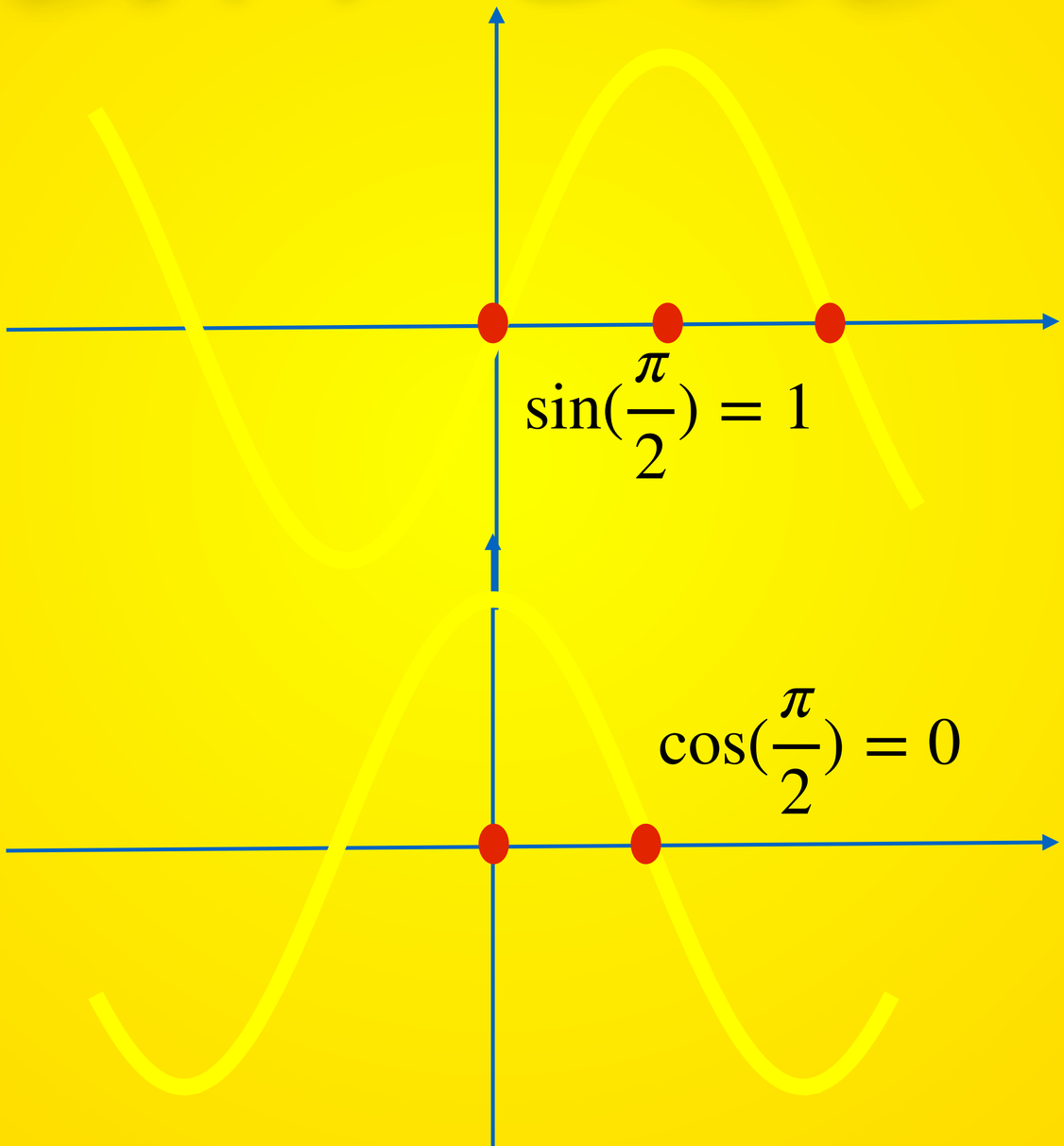
EXAMPLES

- sin
- cos
- tan
- exp
- log
- sqt

- polynomials
- rational functions
- trig polynomial

- arcsin
- arccos
- arctan
- sinc
- sign
- erf

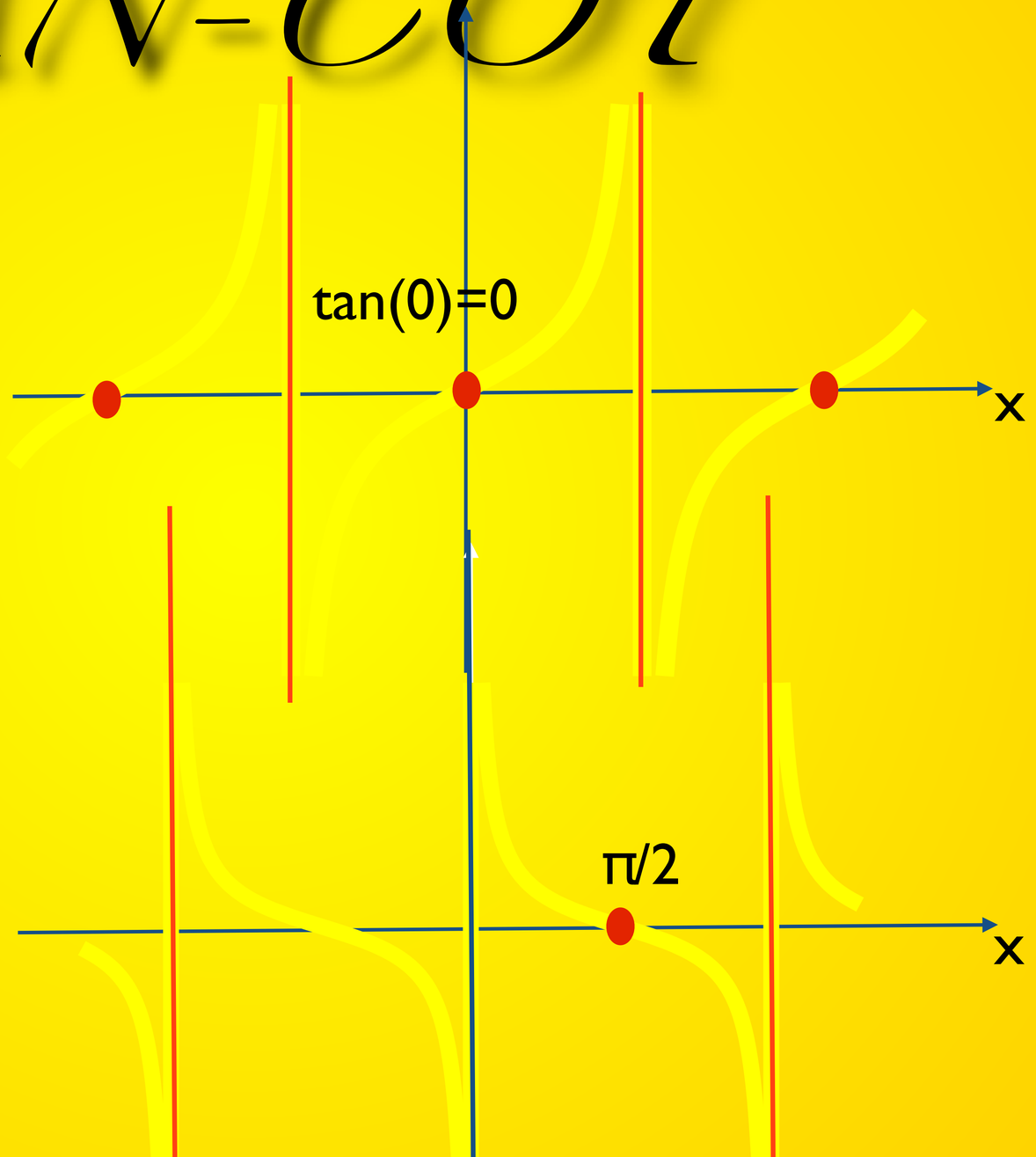
SIN-COS



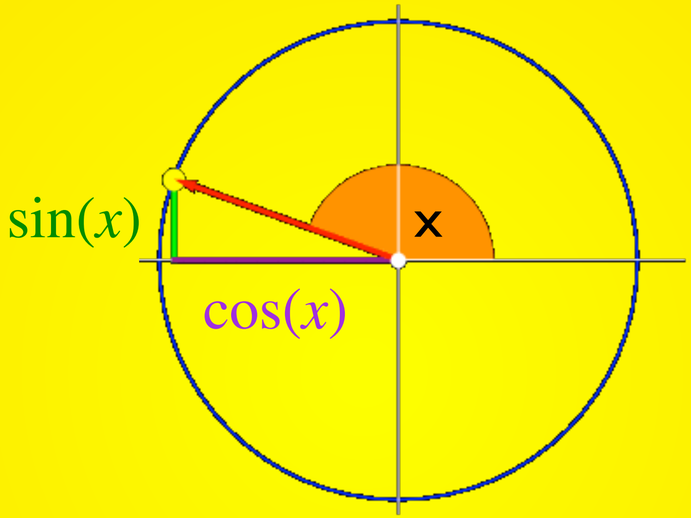
TAN-COT

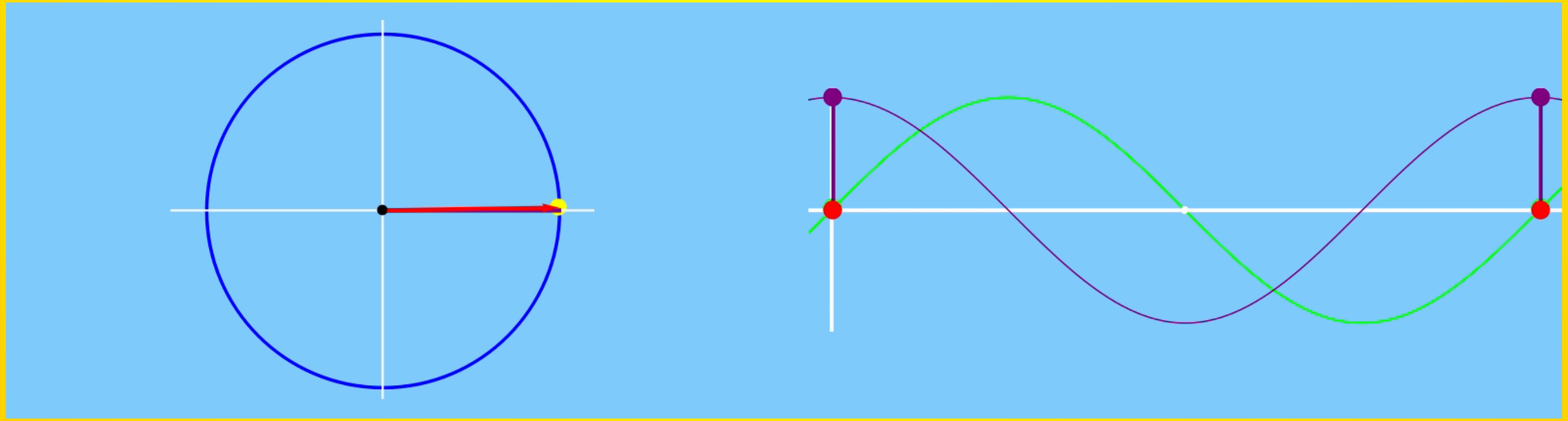
$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$$

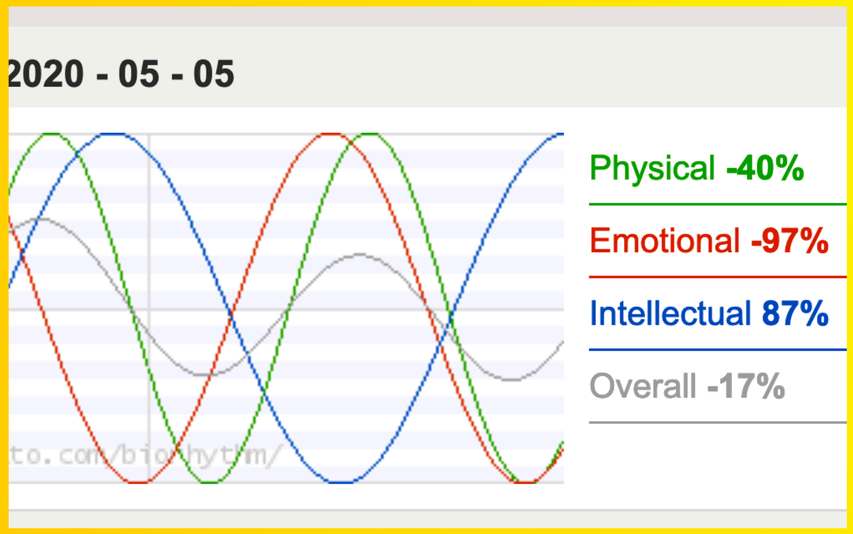


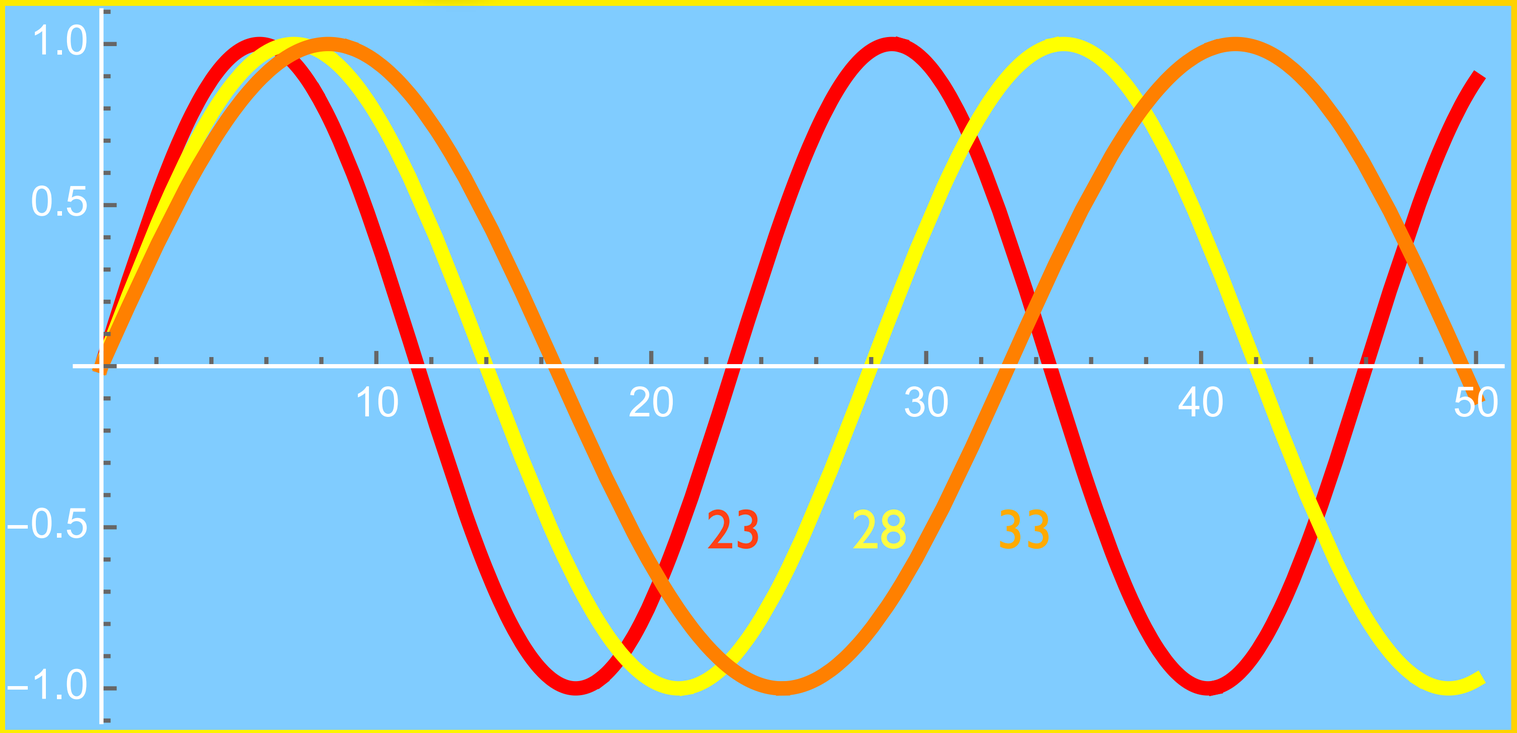
UNIT CIRCLE





BIO-RYTHM





Physical

Emotional

Intellectual

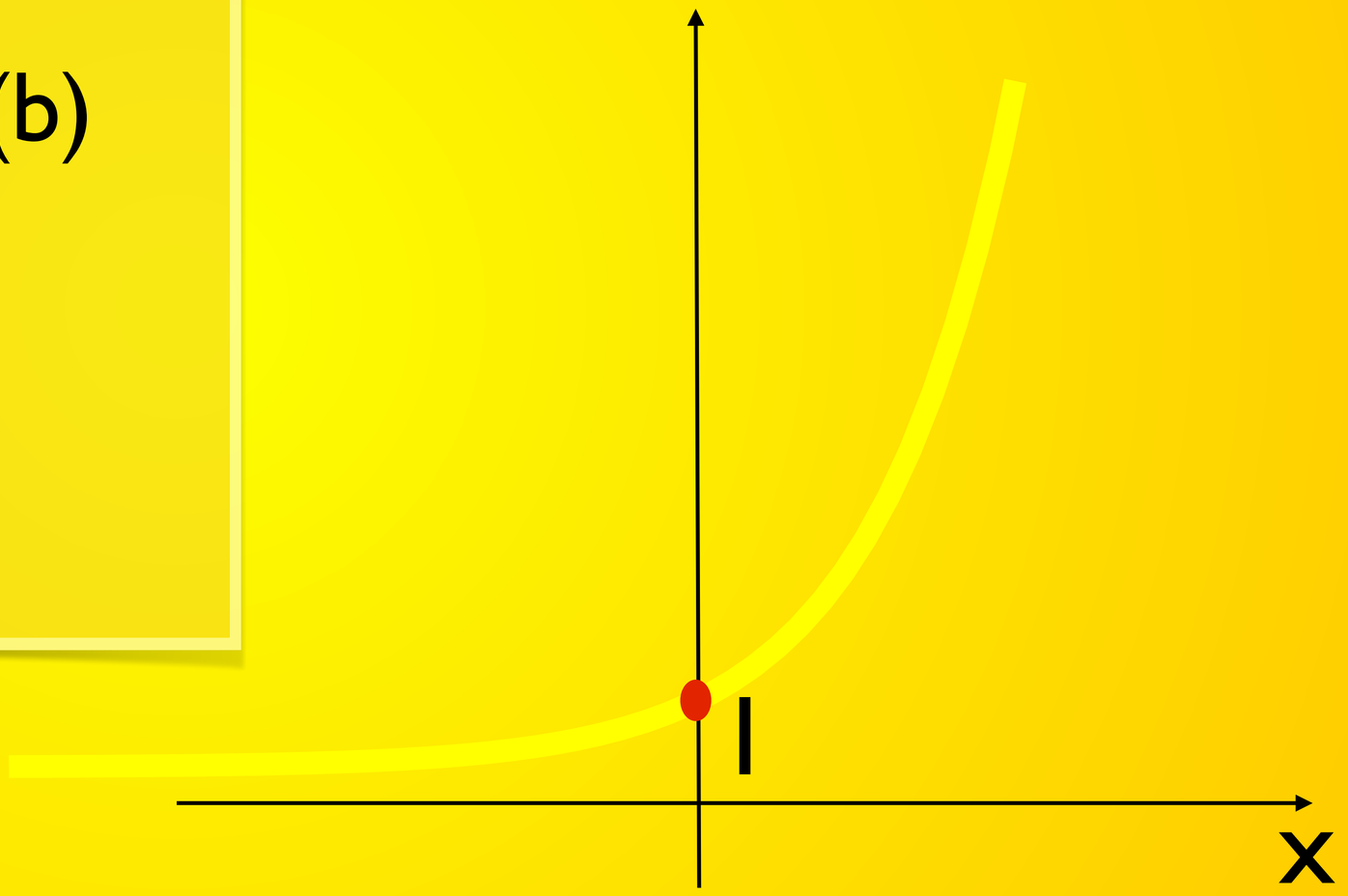
EXP

$$\exp(x) = e^x$$

$$\exp(a+b)=\exp(a)\ \exp(b)$$

$$\exp(0) = 1$$

$$a^b = \exp(b \log(a))$$



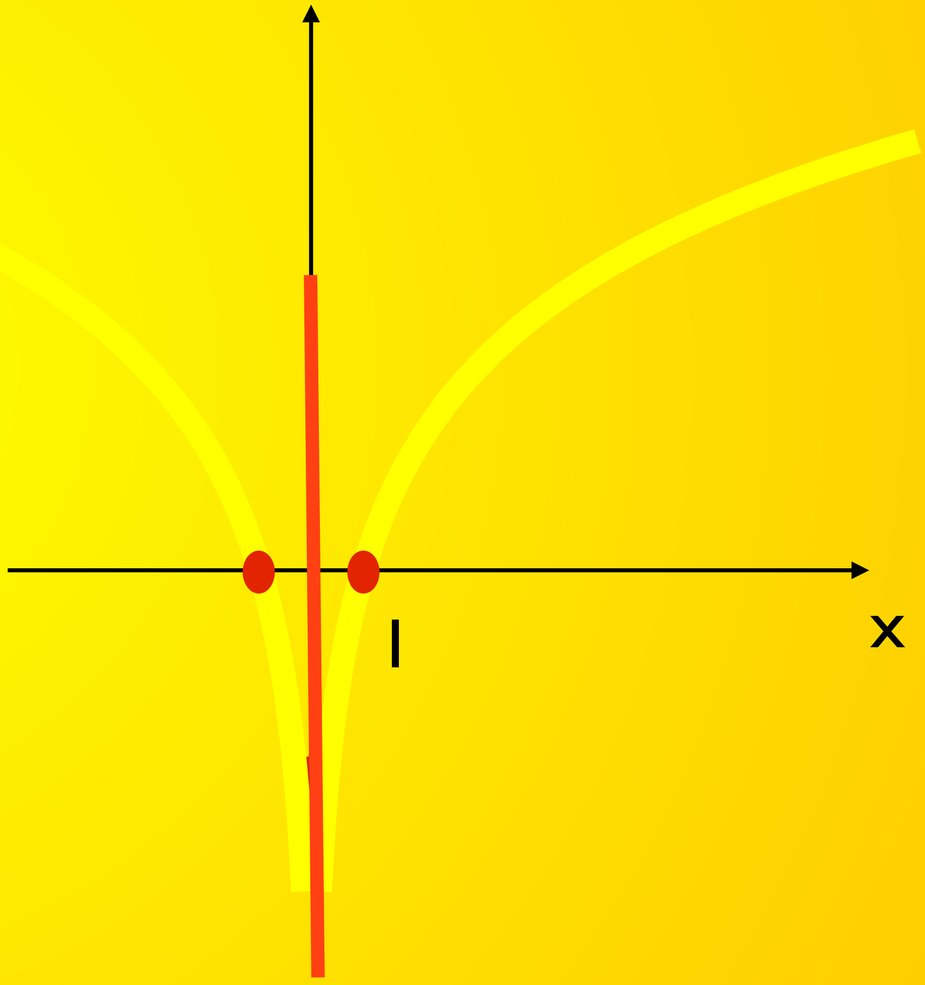
LOG

log|x|=ln|x|

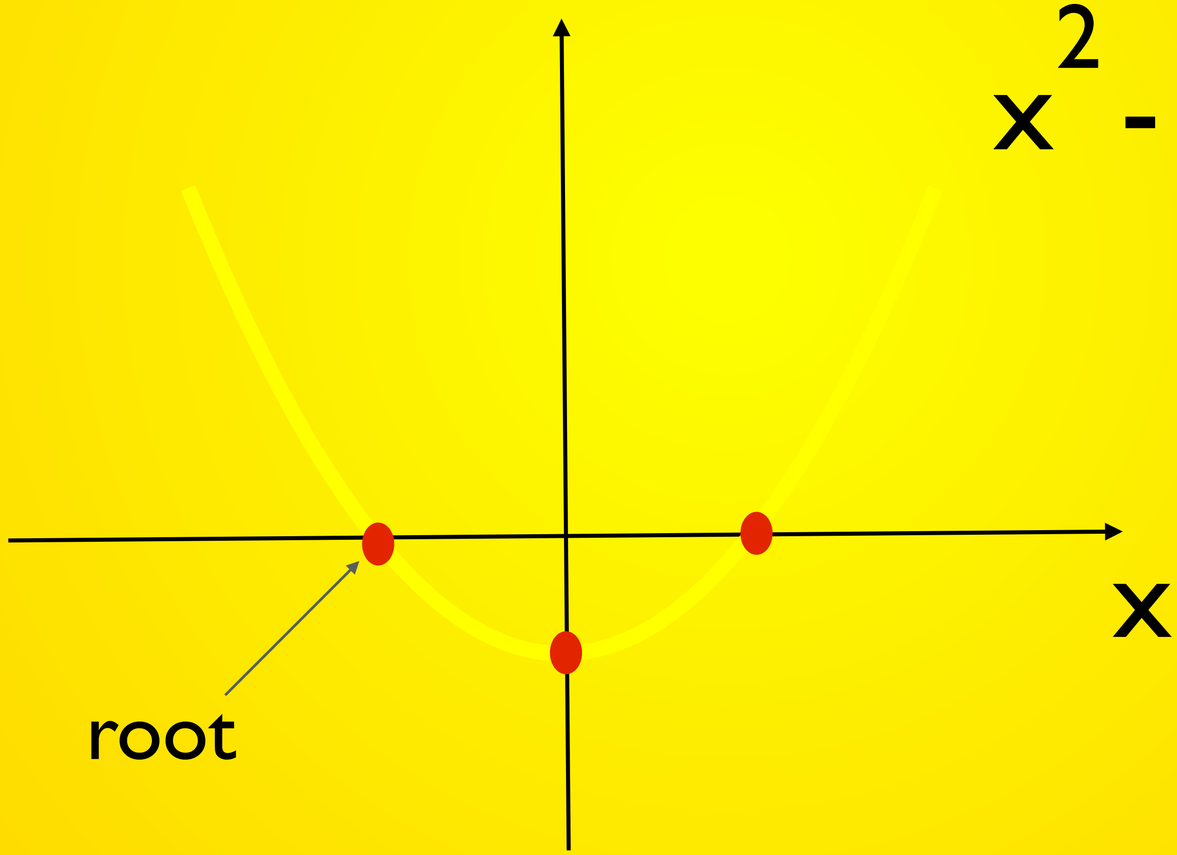
$$\log(1) = 0$$

$$\exp(\log(a)) = a$$

$$\log(a\ b)=\log(a) + \log(b)$$

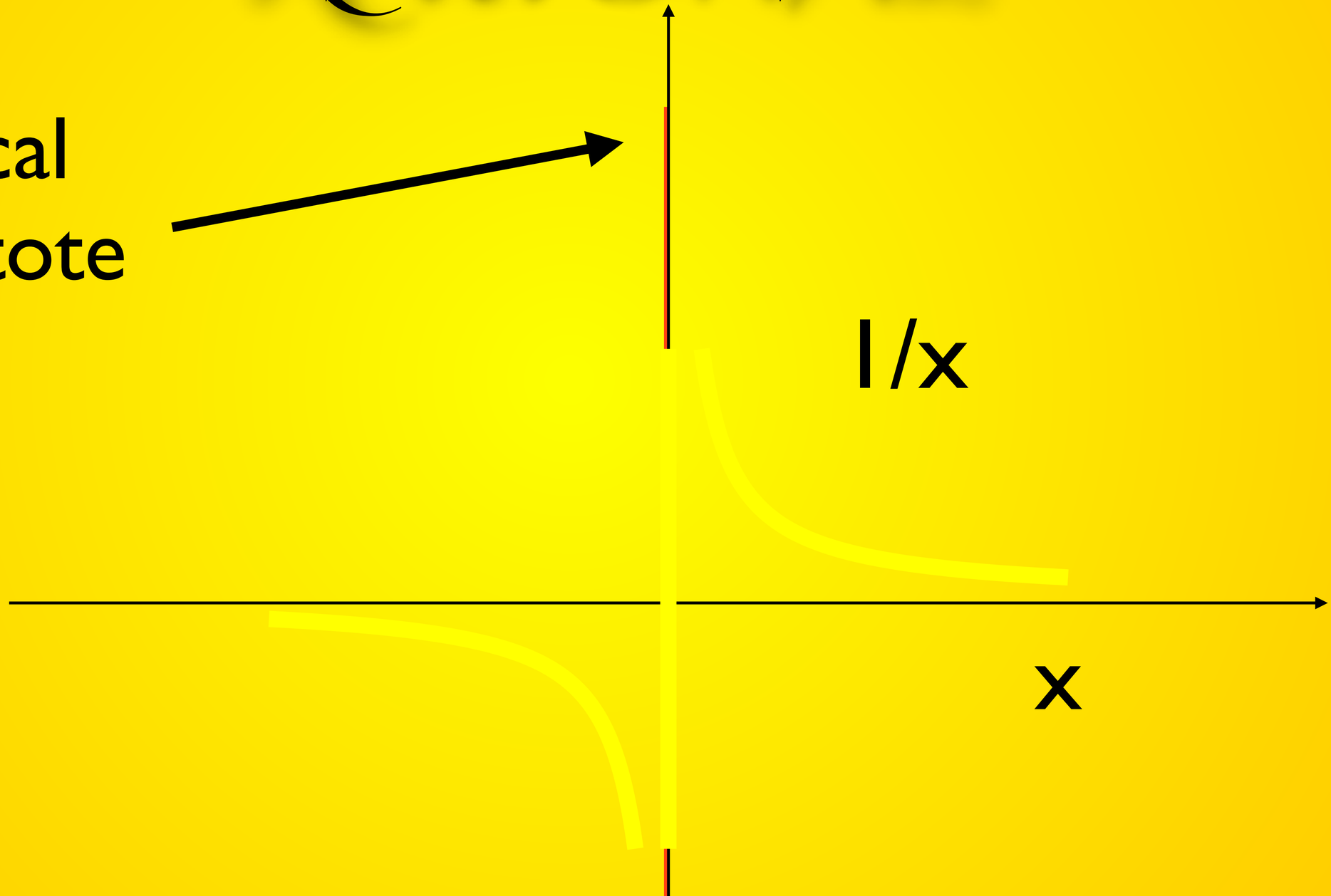


POLYNOMIALS



RATIONAL

vertical
asymptote



ARC TAN

$\pi/2$

arctan(x)

arctan(1)=$\pi/4$

x

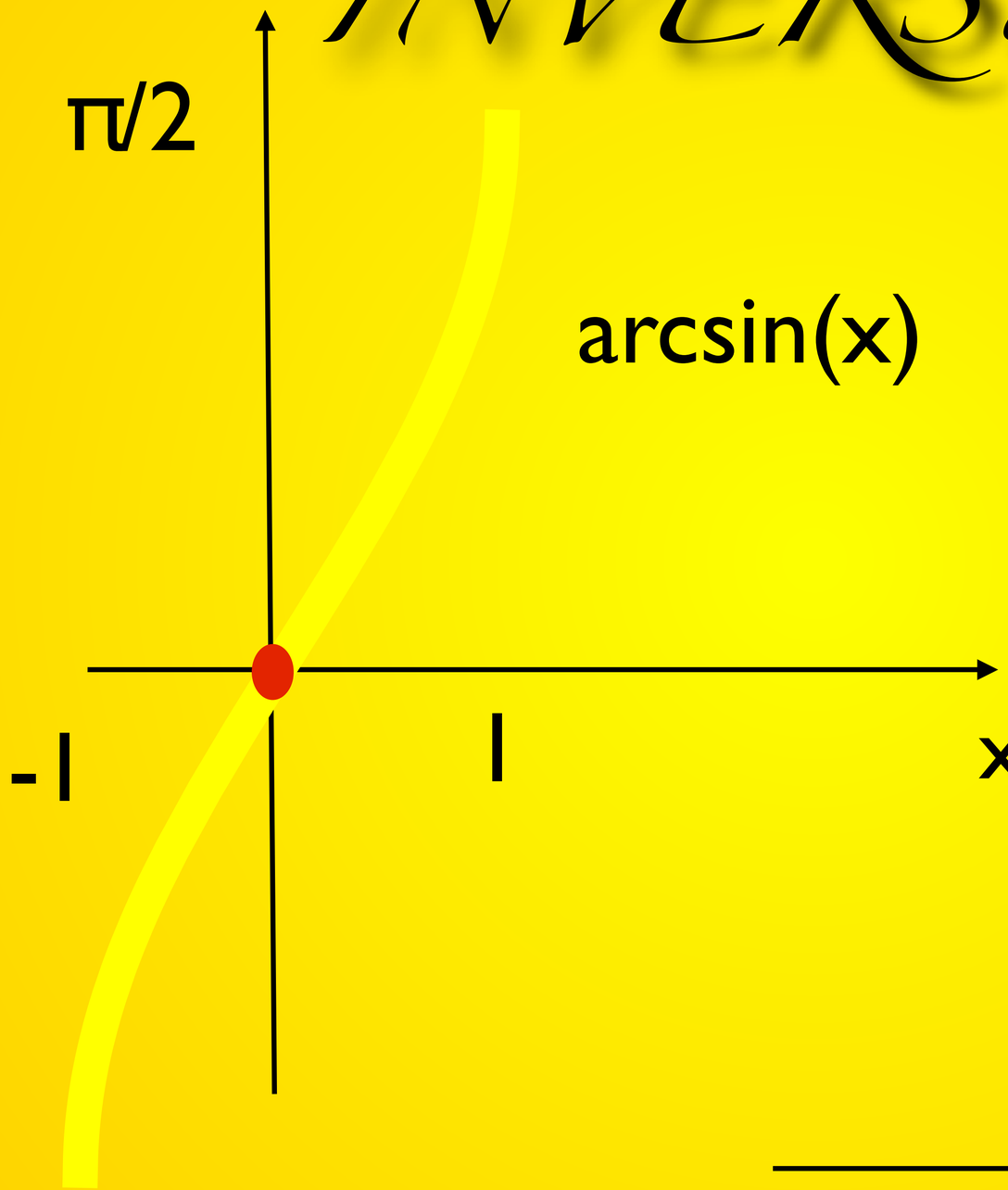
$\pi/2$

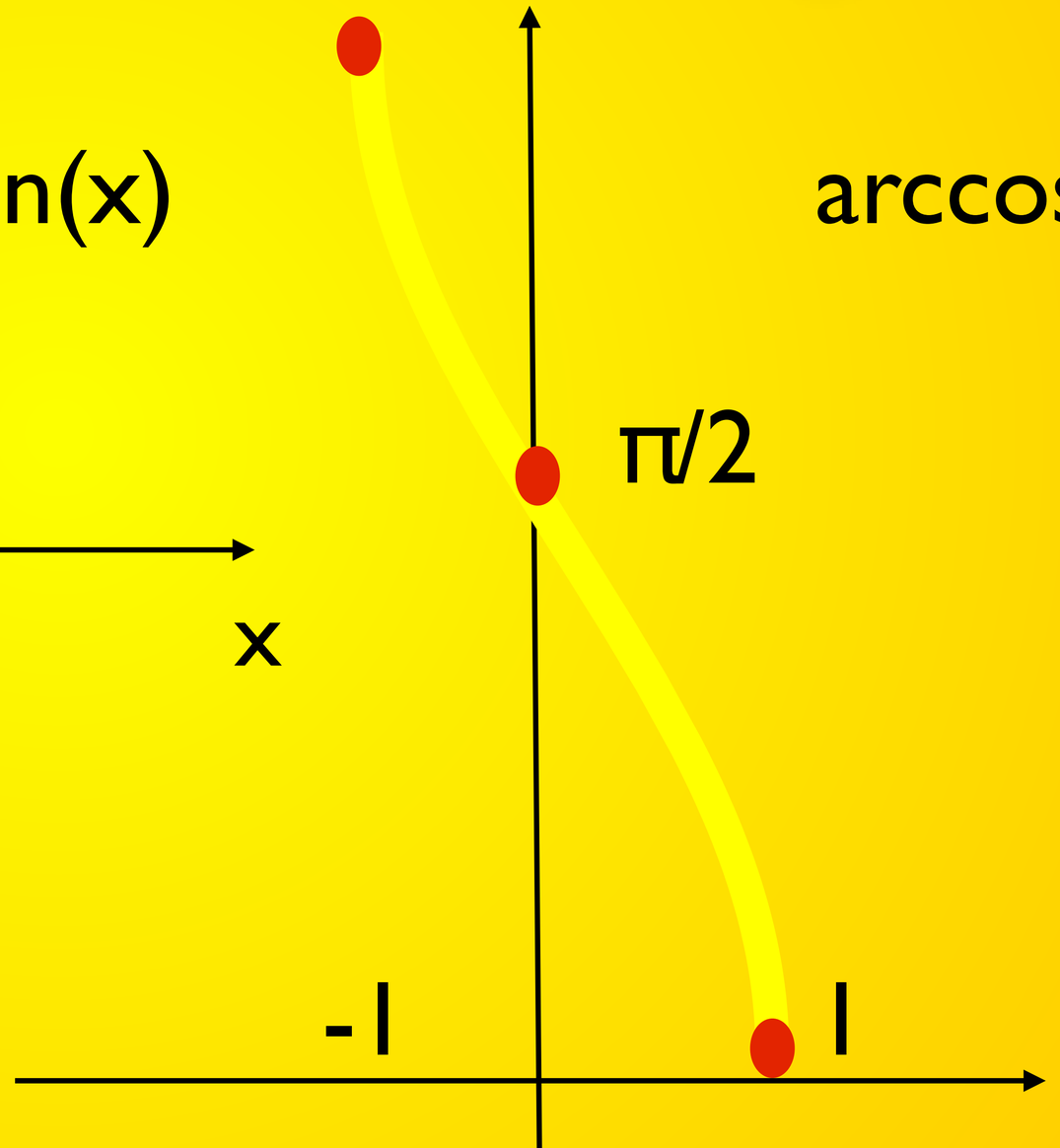
arctan(0.1)
=0.0996687

58 %

tan(Pi/6)

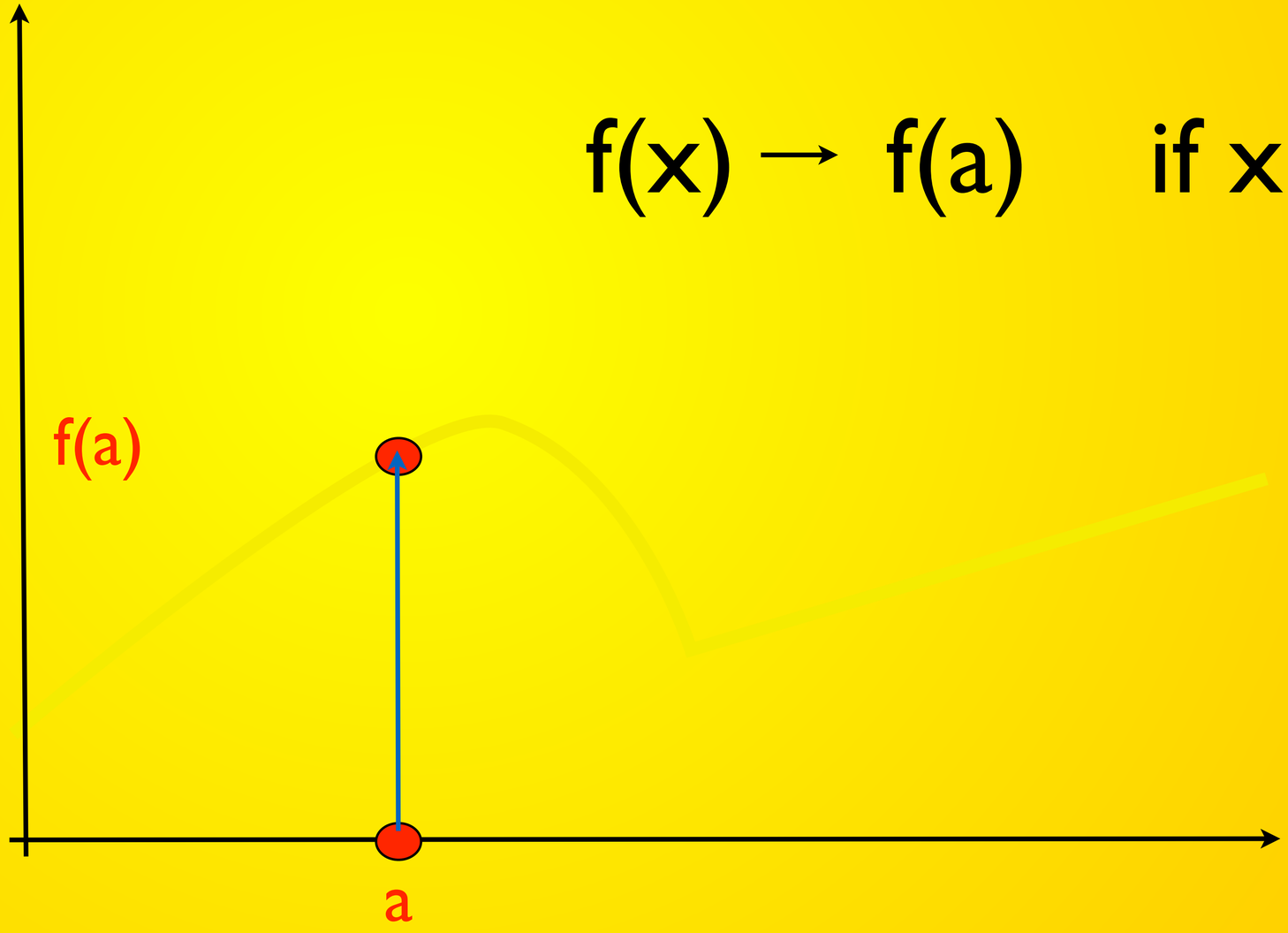
INVERSE TRIG





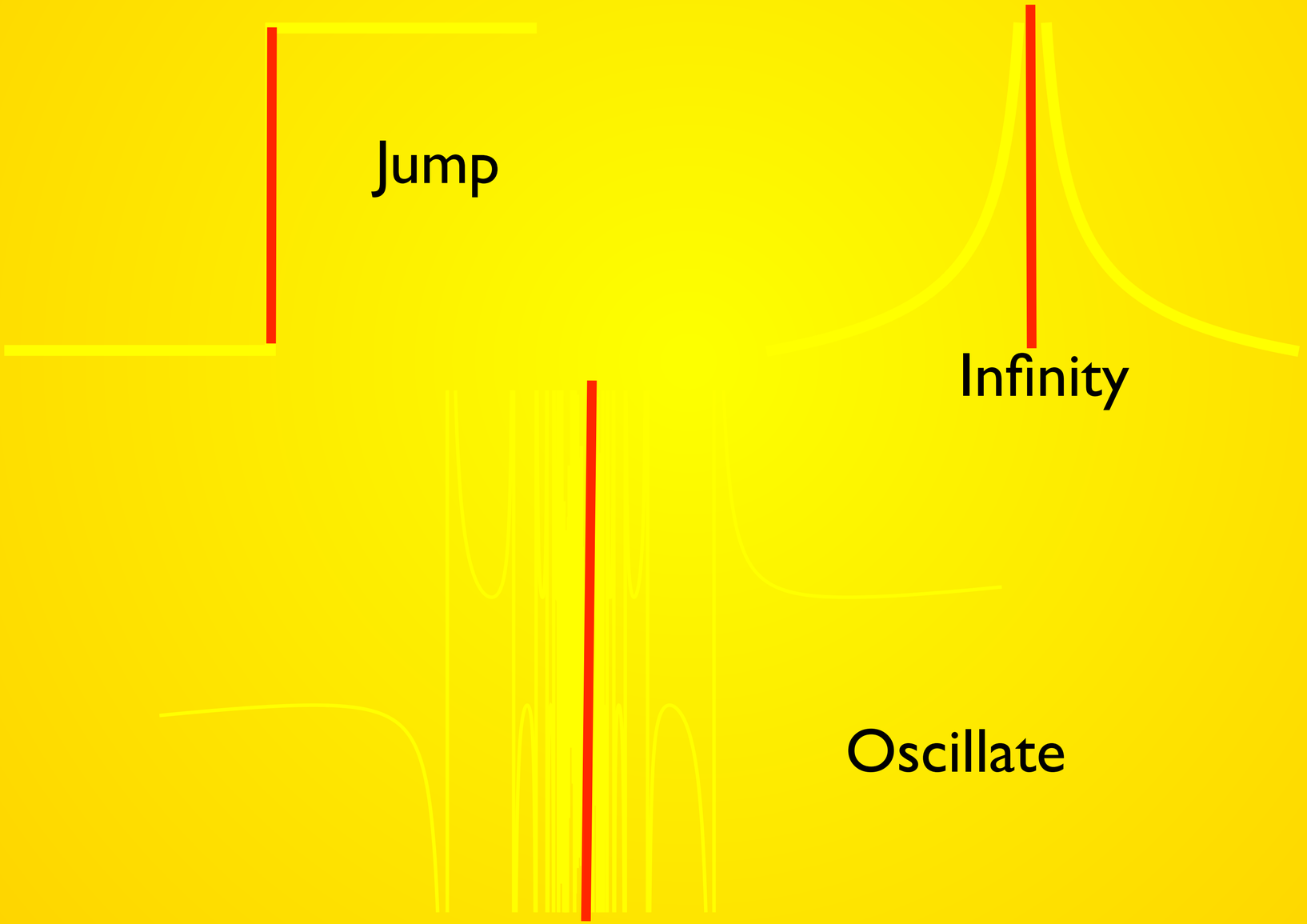
LIMITS

$f(x) \rightarrow f(a)$ if $x \rightarrow a$



Healing Hospital

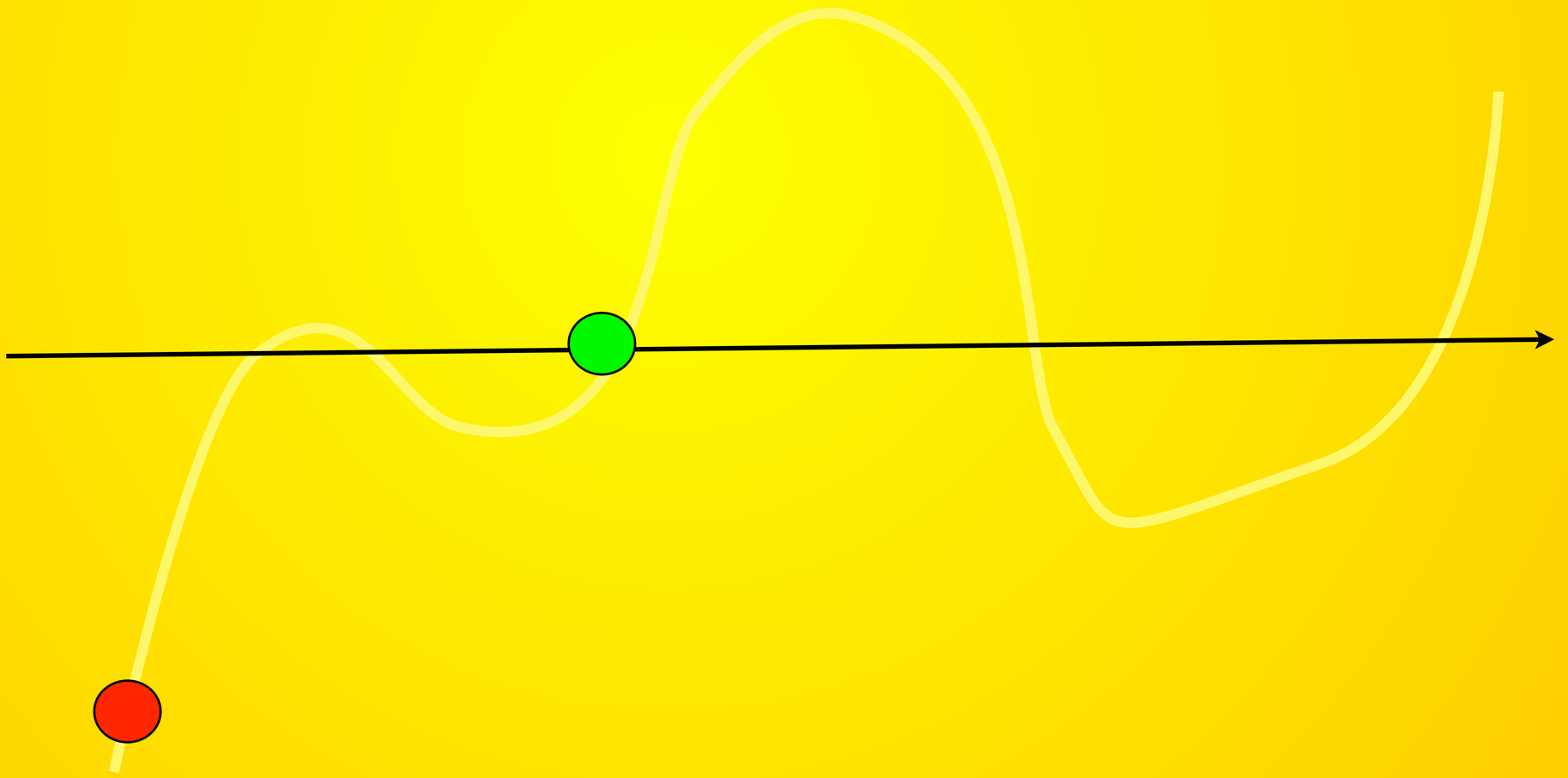
DISCONTINUITIES



THEOREMS

- Intermediate Value theorem
- Fundamental theorem of calc
- Rolle theorem
- Mean value theorem
- Fermat theorem
- Wobbly table theorem
- Strawberry theorem

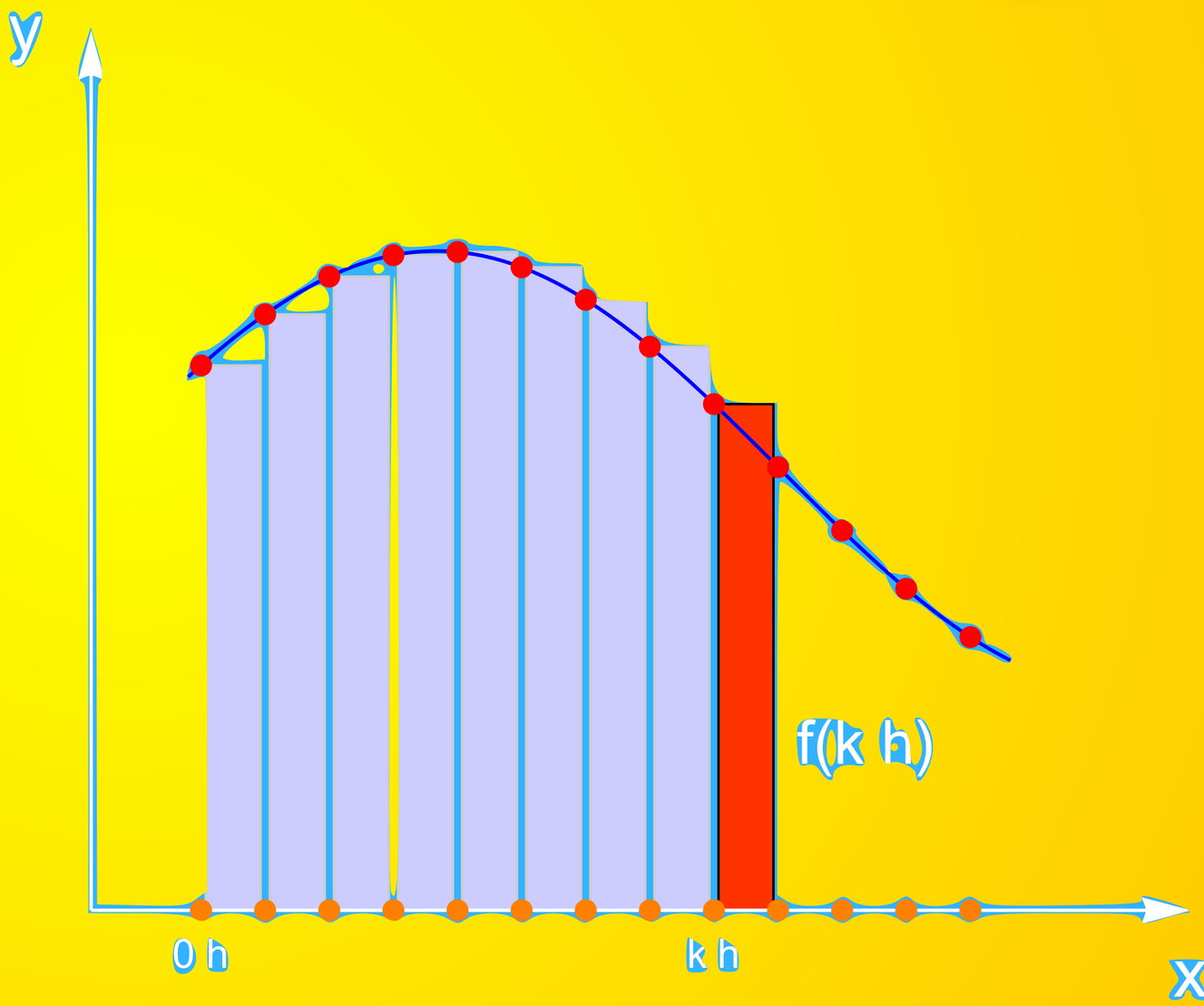
INTERMEDIATE VALUE THEOREM



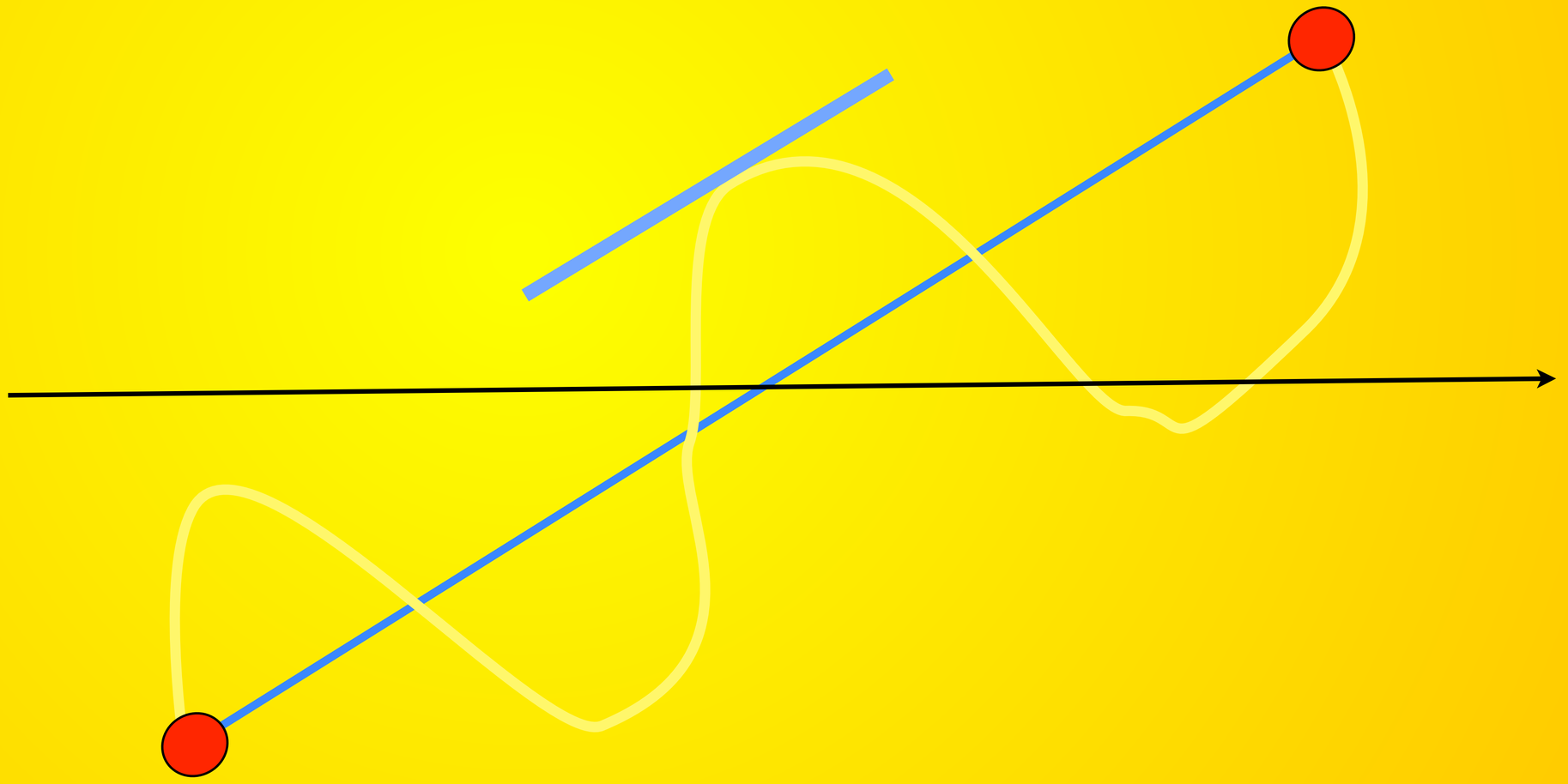
FUNDAMENTAL THEOREM

$$\frac{d}{dx}\int_0^x f(t)dt = f(x)$$

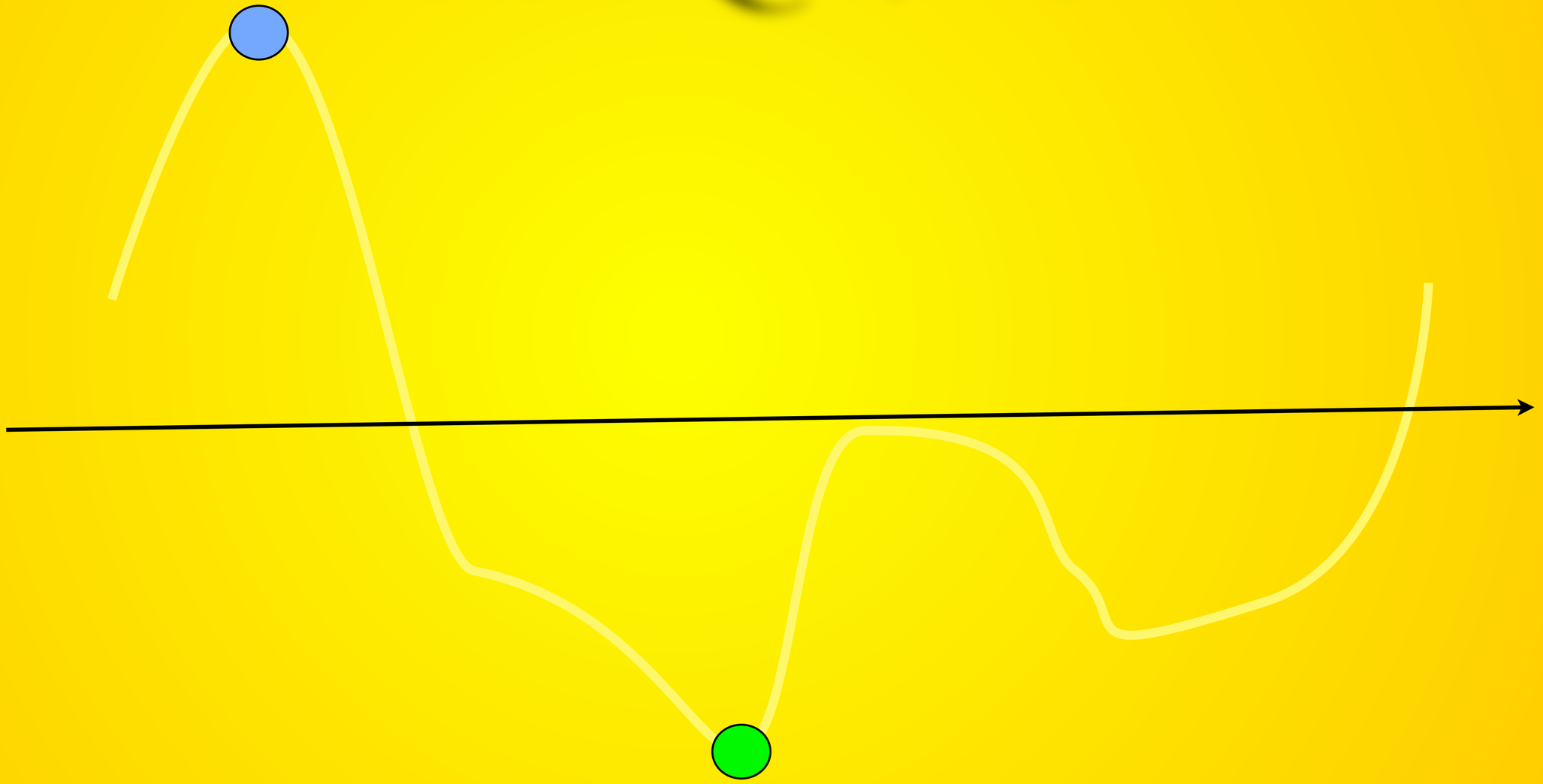
$$\int_0^x f'(t)dt = f(x)$$



MEAN VALUE THEOREM



FERMAT



WOBBLY TABLE

Experiment of Sunday, May 8, 2011

STRAWBERRY

F'=f

g=F/x

f=g and g'=0 have same solutions

DIFFERENTIATION

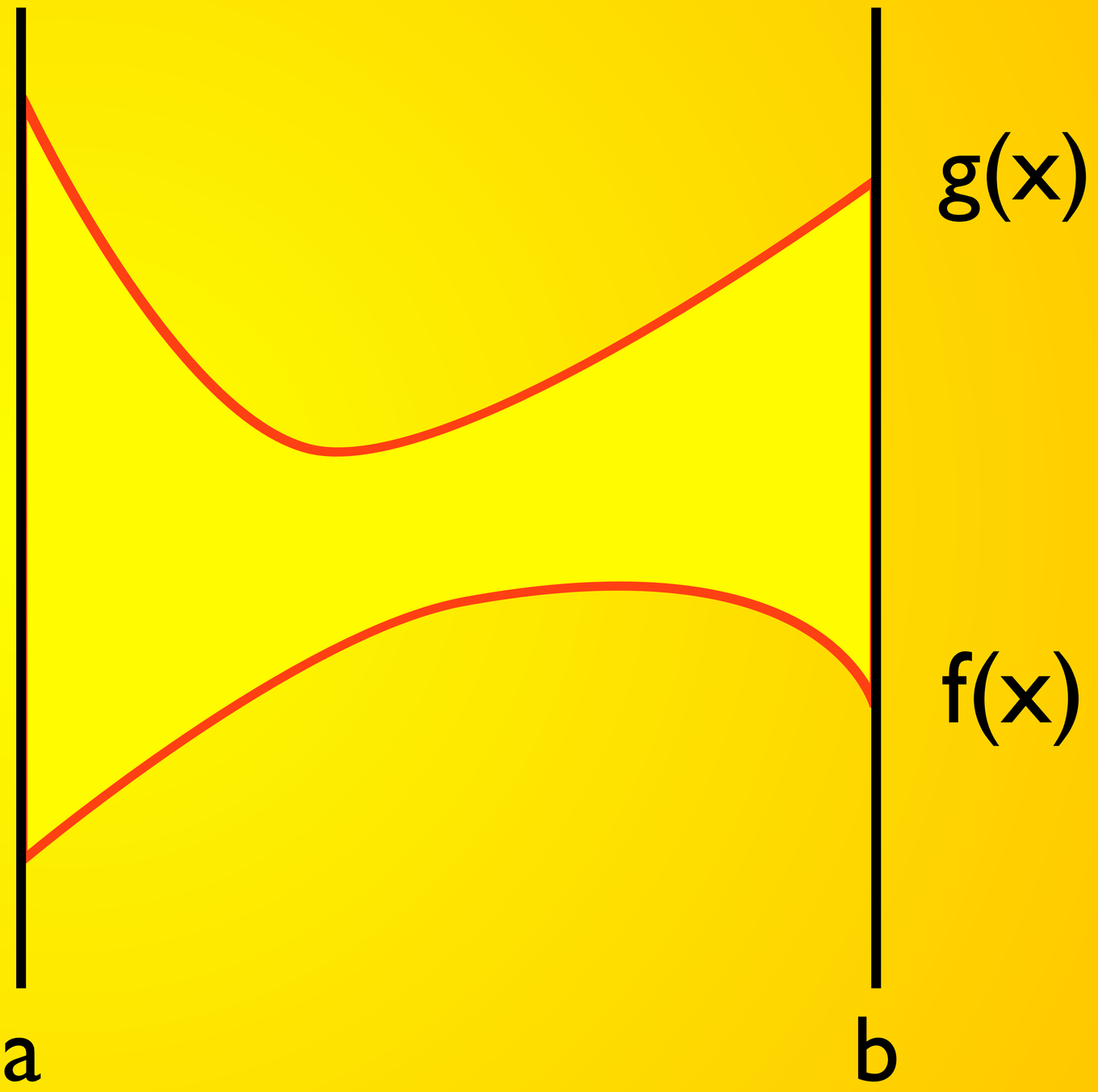
- Product rule $(f\,g)' = f'g+fg'$
- Addition rule $(f+g)' = f'+g'$
- Chain rule $f(g)' = f'(g(x))\,g'(x)$
- Quotient rule $(f/g)' = (f'g-fg')/g$
- Reciprocal rule $(1/f)' = -f'/f$

INTEGRATION

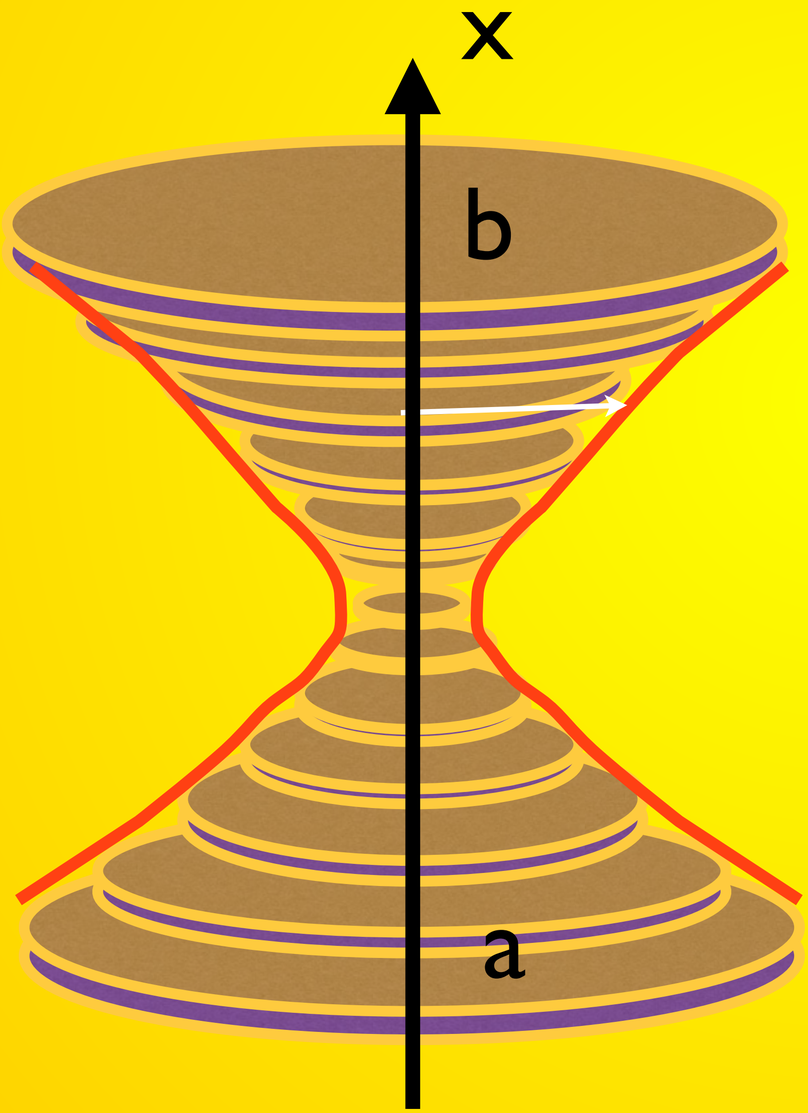
- Substitution $\int f(g(x))\, g'(x)\, dx = \int f(u)\, du$
- Parts $\int uv'dx = uv - \int u' v\, dx$
- Partial Fraction $\frac{1}{(x-a)(x-b)} = \frac{1}{(x-a)} + \frac{1}{(x-b)}$
- Trig Substitution $\int \sqrt{1-x^2}\, dx = \int \cos^2(u)\, du$

AREA

$$\int_a^b g(x) - f(x)dx$$



VOLUME



$$\int_a^b A(x)dx$$

NUMERICAL

- Riemann Sum
- Trapezoid rule
- Simpson Rule
- Monte Carlo

APPLICATIONS

- Statistics: PDF and CDF
- Business: Marginal, total,average cost
- Music: hull , Amplitude
- Computer science:
- AI: building functions
- Economics: Strawberry theorem

- Statistics: PDF and CDF

- Gastronomy: wobbly table
- Psychology: perception catastrophe, solve eqns
- Numerics: Simpson, Newton,Monte Carlo
- Data: fitting, functions

JAM

- In which context did each the following people appear?

l'Hoptial
Fermat
Bolzano
Newton
Mandelbrot
Arnold
Turing

Simpson
Gauss
Feigenbaum
Archimedes
Euler
Thom
Piaget

Harmon
Zeno
Fibonacci
Leibniz
Riemann
Shannon
Benford

Abe Moigno
Gonnik
Archilles
Bolzano
Germaine
Plato
Agnesi

THE END