Lecture 32: Worksheet

This worksheet as well as the solutions was generated by Sofia, a bot written in the academic year 2003/2004 using grant from the Harvard Provost together with Harvard students **Johnny Carlsson**, **Andrew Chi** and **Mark Lezama**. At that time, people have laughed at the chat bot idea. Now it is big business: Google, Siri, Cortana, Wolfram alpha: these are all AI bots which constantly become more and more sophisticated.

1 Differentiate the following functions:
a) $f(x) = 4(x + \tan(x))$
b) $f(x) = x^4 + x$
c) $f(x) = 4(x + \log(x))$

Solution:
a) $f'(x) = 4\left(\sec^2(x) + 1\right)$
b) $f'(x) = 4x^3 + 1$
c) $f'(x) = 4\left(\frac{1}{x} + 1\right)$

2 Integrate the following functions:
a) $f(x) = 3$
b) $f(x) = -3\sin(x)$
c) $f(x) = 1 - e^x$

Solution:
a) $\int f(x) = 3x + C$
b) $\int f(x) = 3\cos(x) + C$
c) $\int f(x) = x - e^x + C$

3 Differentiate the following functions:
a) $f(x) = 0$
b) $f(x) = \frac{3}{\sqrt{x}}$
c) $f(x) = x\log(x)\sec(x)$

Solution:
a) $f'(x) = 0$
b) $f'(x) = -\frac{3}{2x^{3/2}}$
c) $f'(x) = \sec(x)(\log(x) + x\log(x)\tan(x) + 1)$

4 Integrate the following functions:

a) $f(x) = 30x^4\sqrt{x^5}$

b) $f(x) = 3\left(\frac{1}{x^2} + e^x + 1\right)$

c) $f(x) = -e^{-x}(x-2)x$

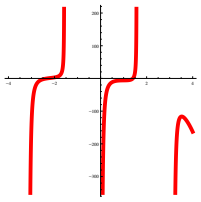
Solution:

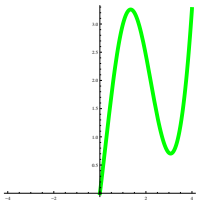
a) $\int f(x) = 4\left(x^5\right)^{3/2} + C$

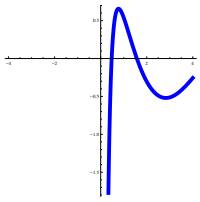
b) $\int f(x) = 3\left(x + e^x - \frac{1}{x}\right) + C$

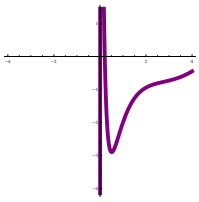
c) $\int f(x) = e^{-x}x^2 + C$

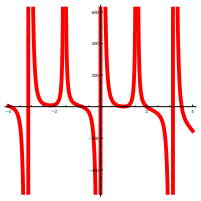
5 Match the following functions with derivatives:

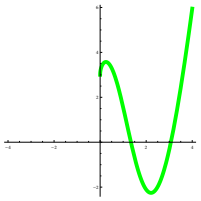
a) 

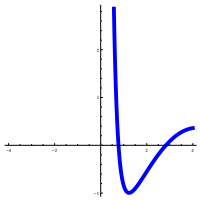
b) 

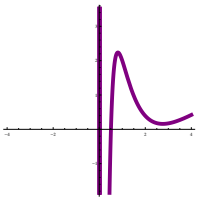
c) 

d) 

1) 

2) 

3) 

4) 

Solution:

a → 1, b → 2, c → 3, d → 4

6 Find the critical points of the following functions:

a) $f(x) = (x-6)^2$

b) $f(x) = (x-8)(x-7)$

c) $f(x) = (x-6)(x-5)^2$

Solution:

a) $f'(x) = \{\{x \to 6\}\}$

b) $f'(x) = \left\{\left\{x \to \frac{15}{2}\right\}\right\}$

c) $f'(x) = \left\{\{x \to 5\}, \left\{x \to \frac{17}{3}\right\}\right\}$