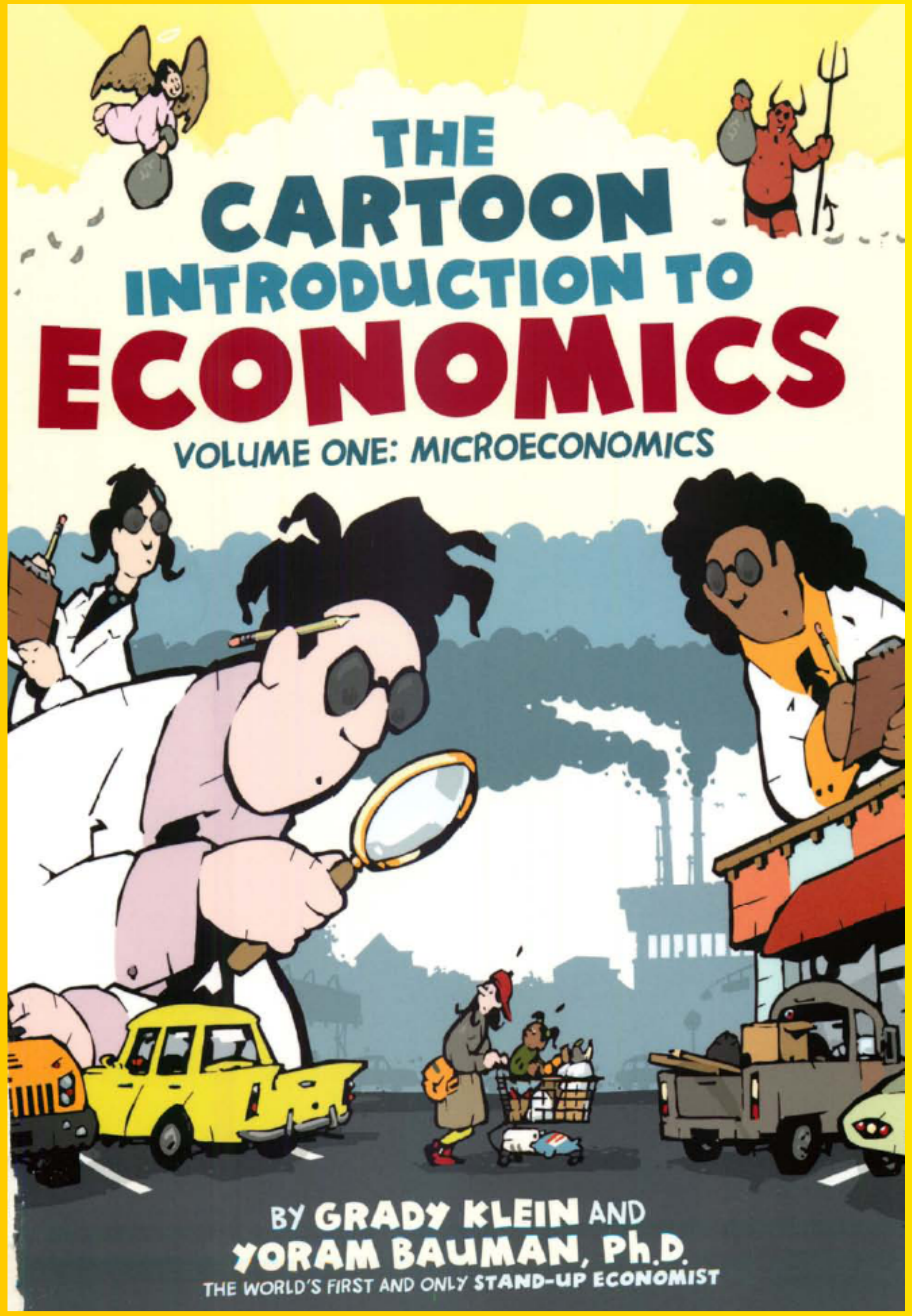
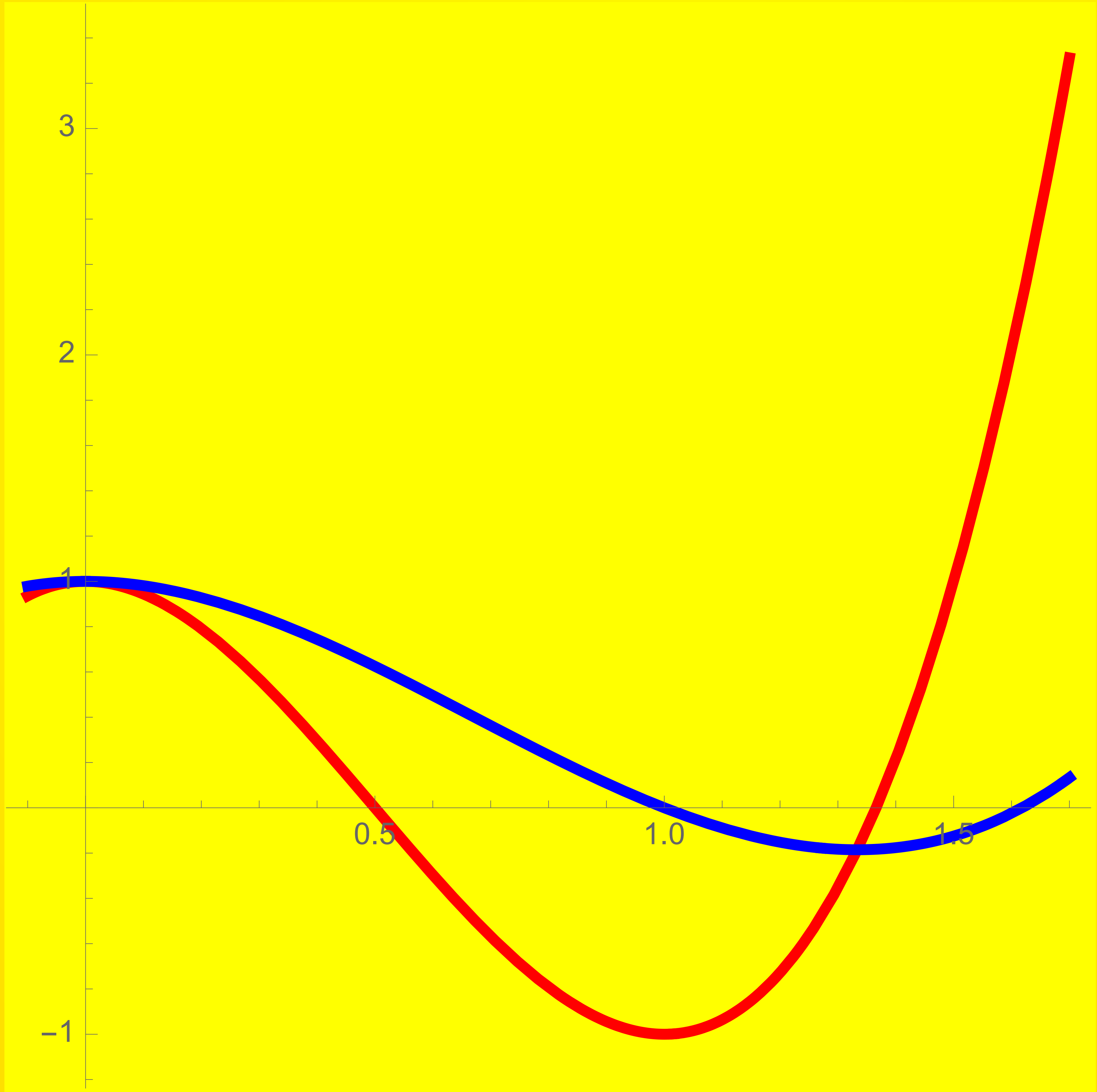
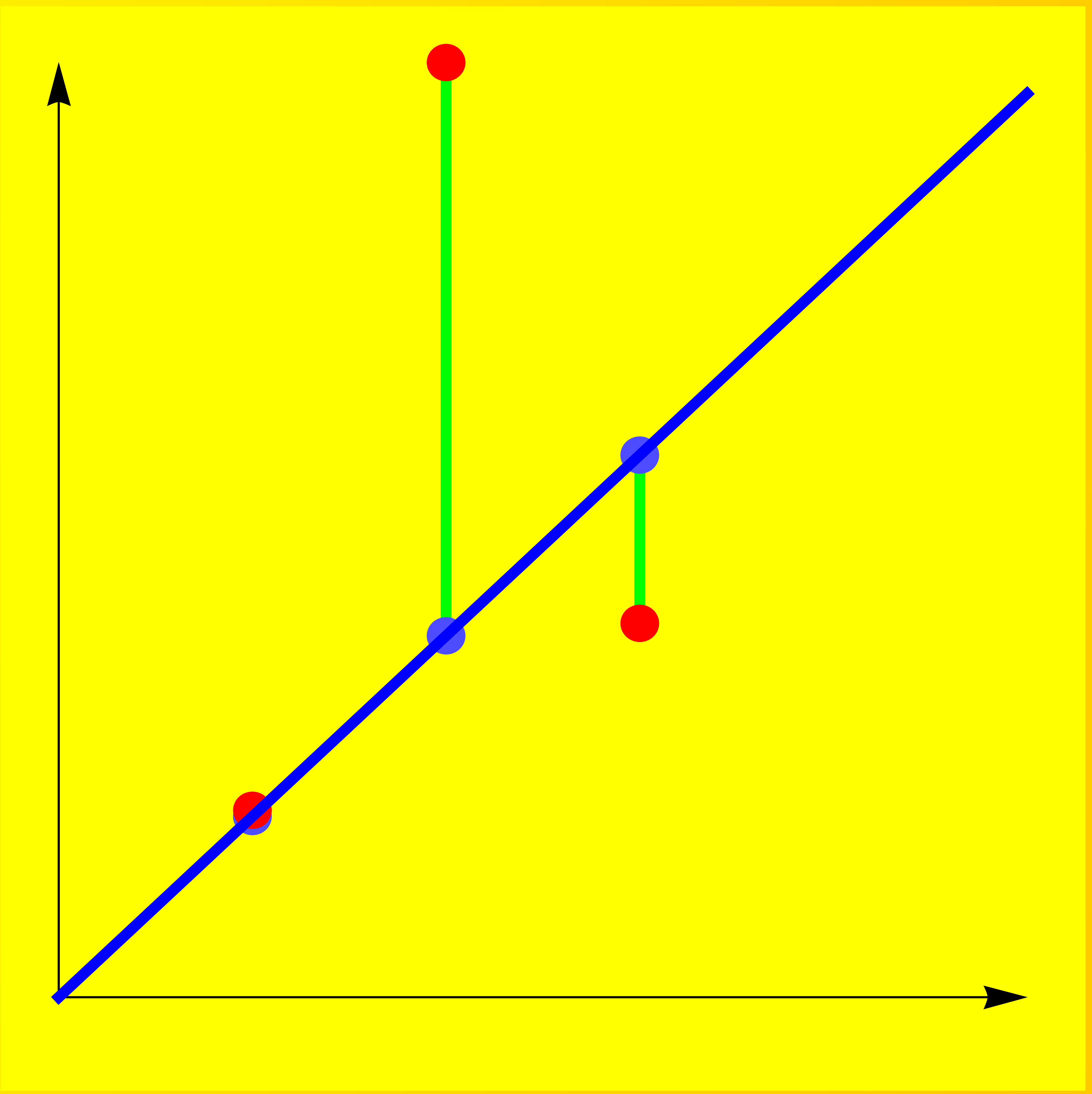
31

Economics







PLAN

1. Poll

2. Costs

3. Strawberry Theorem

4. Data fitting

5. Example

6. Jam

POLL

What does "marginal cost" mean?

a) Excess cost for production

b) Change in cost in dependence of time

c) Change of cost in dependence of goods http://www.efzg.hr/jsohinger_en

Here is a very short glossary of some economic concepts and their explanations in mathematics (calculus). Thanks to the economist Jasminka Sohinger to suggest and explained to me (Oliver) some of the terms. (Any possible errors are of course mine):

JARGON

Term	Explanation
Cobb-Douglas function	$f(x,y) = A\ x^{alpha}\ y^{beta}$, example of an utility function. x,y are usually referred as goods or bundles of goods
Budget constraint	special constraint g(x,y) = c, the constant c is the level of income
Utility function	A function of two variables f(x,y) measuring the level of utility, the variables x,y typically represent goods. There can be more variables.
Production function	A function of two variables f(x,y) measuring the level of production, x,y typically are labor and capital. There can be more variables.
Cost function	A function of two variables f(x,y), where x,y are inputs The function can be a constraint. There can be more variables.
Objective function F	Function of several variables f, to be maximized or minimized.
Program	Problem
Duality	Maximizing utility given expenditures or minimizing expenditures by fixing utility. Example in mathematics: minimize the surface area under fixed volume or maximize the volume when surface area is fixed.
Convex sets	A set Y is convex if for all a in [0,1], and all x,y in Y also a x + (1-a) y is in Y. Economists prefer convexity since in decision processes, one wants to interpolate between two points x,y.
Convex function	A function is **convex** on a convex set S if f(a*x + (1-a) y) smaller or equal than a*f(x) + (1-a)*f(y) for all a in [0,1] and all x,y in S. Example: $f(x,y) = x^2+y^2$ is convex. A function is **concave** on S, if -f is convex on S.
Quasiconvex function	An utility function f(x,y) is quasiconvex if the lower level sets Y_c = { f(x,y) smaller or equal than c } are convex sets. Example: f(x,y) = -xy is quasiconvex on the first quadrant S. Note that it is not convex because 1 = f((1,1)) = f(1/2((0,0) + (2,2))) is smaller than 1/2 [f((0,0)) + f (2,2))] = 2.
Quasiconcave function	Utility function f(x,y) is quasiconcave if -f is quasiconcave. Equivalently, the **upper** level sets Y_c = { f(x,y) larger or equal than c } are convex sets. Cobb-Douglas functions like $f(x,y) = 3\ x^2\ y^3$ are quasiconcave. (*)
Programming problem:	Find the global minimum f on the region G defined by $h_i=0$, g_j less or equal to 0
Linear programming problem:	Programming problem, where f and g_i,h_j are all linear.
Convex programming:	The function f and the contraints g_j are all convex
Quadratic programming:	Contraints are linear, the objective function is a sum of a linear function and a quadratic form.
Kuhn-Tucker conditions	Conditions for global minimum: convex f,g_i, affine h_j

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PROJECT

Pi and Mandelbrot

Costs

x number of goods
like berries

F(x) total cost
f(x)=F'(x) marginal cost
g(x)=F(x)/x average cost

f(x) cost of
berry number x.

F(x) cost of x berries

Strawberries

Strawberry Theorem

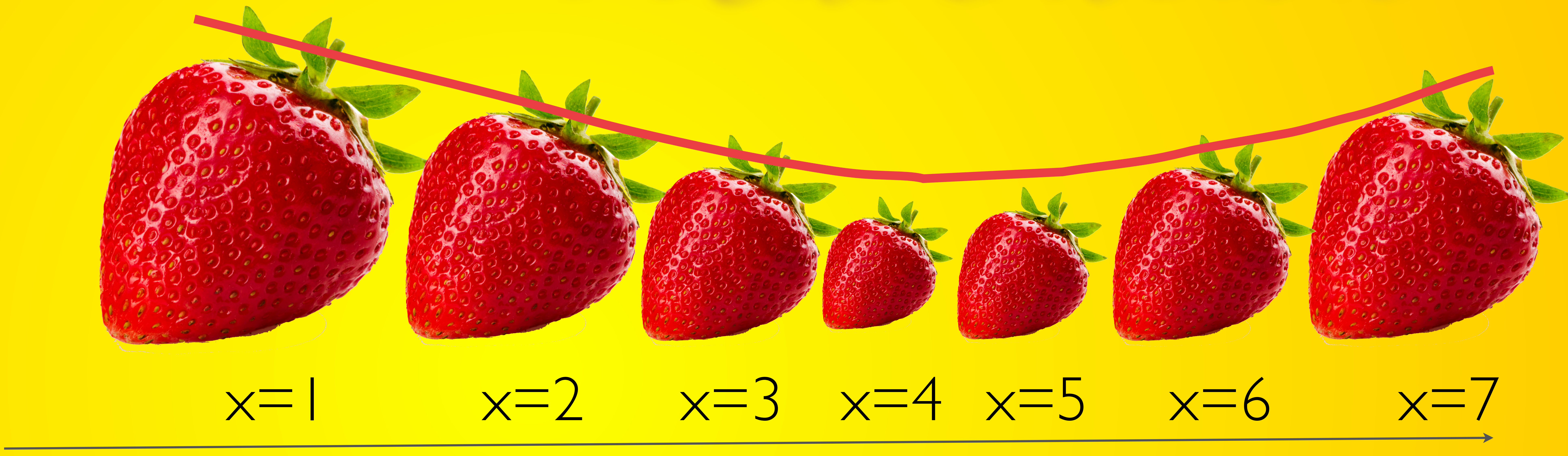
$$f = g \Leftrightarrow g' = 0$$

F(x) total cost
f(x)=F'(x) marginal cost
g(x)=F(x)/x average cost

"Critical points of the average cost are break even points"

see Heckner/Kretschmer:
"Don't worry about Micro"

Illustration



Total Cost F(3)

marginal cost f(3)

Proof

$$g' = \left(\frac{F}{x}\right)' = \frac{F'}{x} - \frac{F}{x^2} = \frac{1}{x}\left(F' - \frac{F}{x}\right) = \frac{1}{x}(f - g)$$

QED

F(x) total cost, f(x)=F'(x) marginal cost,
g(x)=F(x)/x average cost

Source

see Heckner/Kretschmer:
"Don't worry about Micro"

9.5 The Connection Between Marginal and Average Costs

This section is slightly more technical than the rest of this chapter. The subject of our analysis at this point is the connection between different cost curves. To be more precise, we investigate how the MC curve cuts through the average cost curves at their respective minima. This is shown in Fig. 9.4.

Shutdown and Breakeven Points

Before we commence with our analysis, let us link the graph back to some of the discussion above. Looking at Fig. 9.4 you may have noticed that the two quantities for the intersection of MC with AVC and MC with ATC are labelled Q_S and Q_B. These are the shutdown and breakeven points, respectively.

> The MC curve crosses the AVC and ATC curves at their respective minima.

As we discussed previously, when employing the profit-maximising condition and when AVC is equal to MC, we would be just about indifferent between shutting down or producing in the short run. Secondly, when we set price equal to MC and when at this point MC is also equal to ATC, the firm is just breaking even.

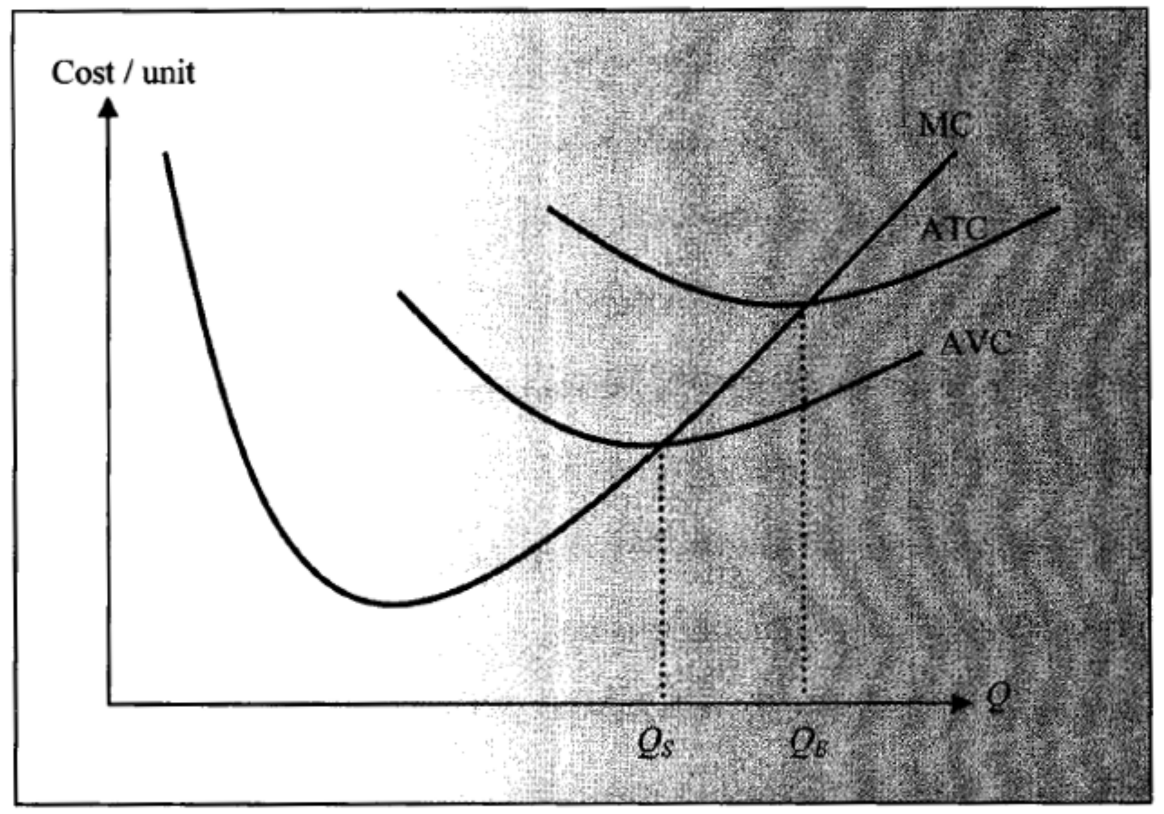


Fig. 9.4. The marginal cost MC curve cuts through average variable cost AVC and average total cost ATC curves at their respective minima. These points are the shutdown and break-even points, respectively

You take one strawberry after another and place them on a scale that tells you the average weight of all strawberries. The first strawberry that you place in the bucket is very large, while every subsequent one will be smaller (until you reach the smallest one). Because of the literal "weight" of the heavier ones, average weight is larger than marginal weight (i.e. the weight of each strawberry you handle). Average weight still decreases, although less steeply than marginal weight.

Once you reach the smallest strawberry, every subsequent strawberry will be larger, which means that the rate of decrease of the average weight becomes smaller and smaller until eventually it stands still. At this point, the marginal weight is just equal to the average weight.

Mathematical Proof

Rather than blindly trusting the intuition above, we can also prove our analysis mathematically. Let us perform this proof for the intersection of MC and ATC. Our first step is to compute the derivative of ATC with respect to Q and set this equal to zero to find the curve's critical point, here the minimum:

$$\frac{d\text{ATC}}{dQ} = 0 \tag{9.14}$$

In order to make Equation 9.14 usable, let us substitute TC/Q for ATC. Therefore, we get:

$$\frac{d(\frac{\text{TC}}{Q})}{dQ} = 0 \tag{9.15}$$

To avoid complicated calculus, let us reformulate the numerator as a product:

$$\frac{d(\text{TC} \cdot Q^{-1})}{dQ} = 0 \tag{9.16}$$

Remembering the **product rule**, we differentiate TC $\cdot$ Q^{-1} with respect to Q by taking the derivative of the first term and multiplying it by the second, and adding the derivative of the second term and multiplying it by the first. This gives us:

$$\frac{d\text{TC}}{dQ} Q^{-1} - \text{TC} \cdot Q^{-2} = 0 \tag{9.17}$$

Notice the negative sign between the terms, which is a result of the –1 "brought down" from Q^{-1} of Equation 9.16 in the process of differentiation. When looking at Equation 9.17, we notice that the very first term is MC and so we can write:

$$\text{MC} \cdot Q^{-1} - \text{TC} \cdot Q^{-2} = 0 \tag{9.18}$$

As a final step we multiply both sides by Q and write the second term as a fraction:

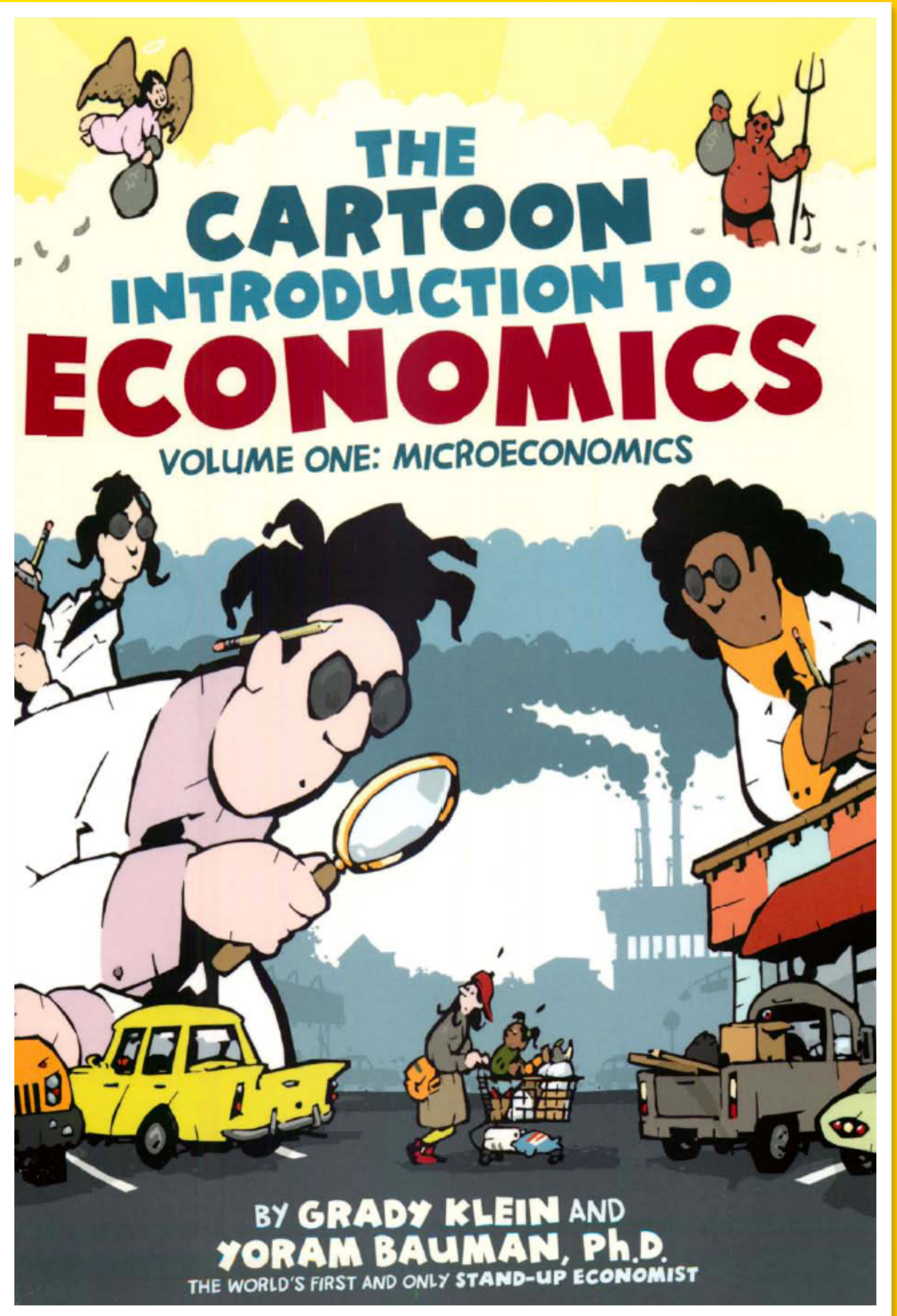
$$\text{MC} - \frac{\text{TC}}{Q} = 0 \tag{9.19}$$

Since, by definition, TC/Q is equal to ATC, we finalise our equation to become:

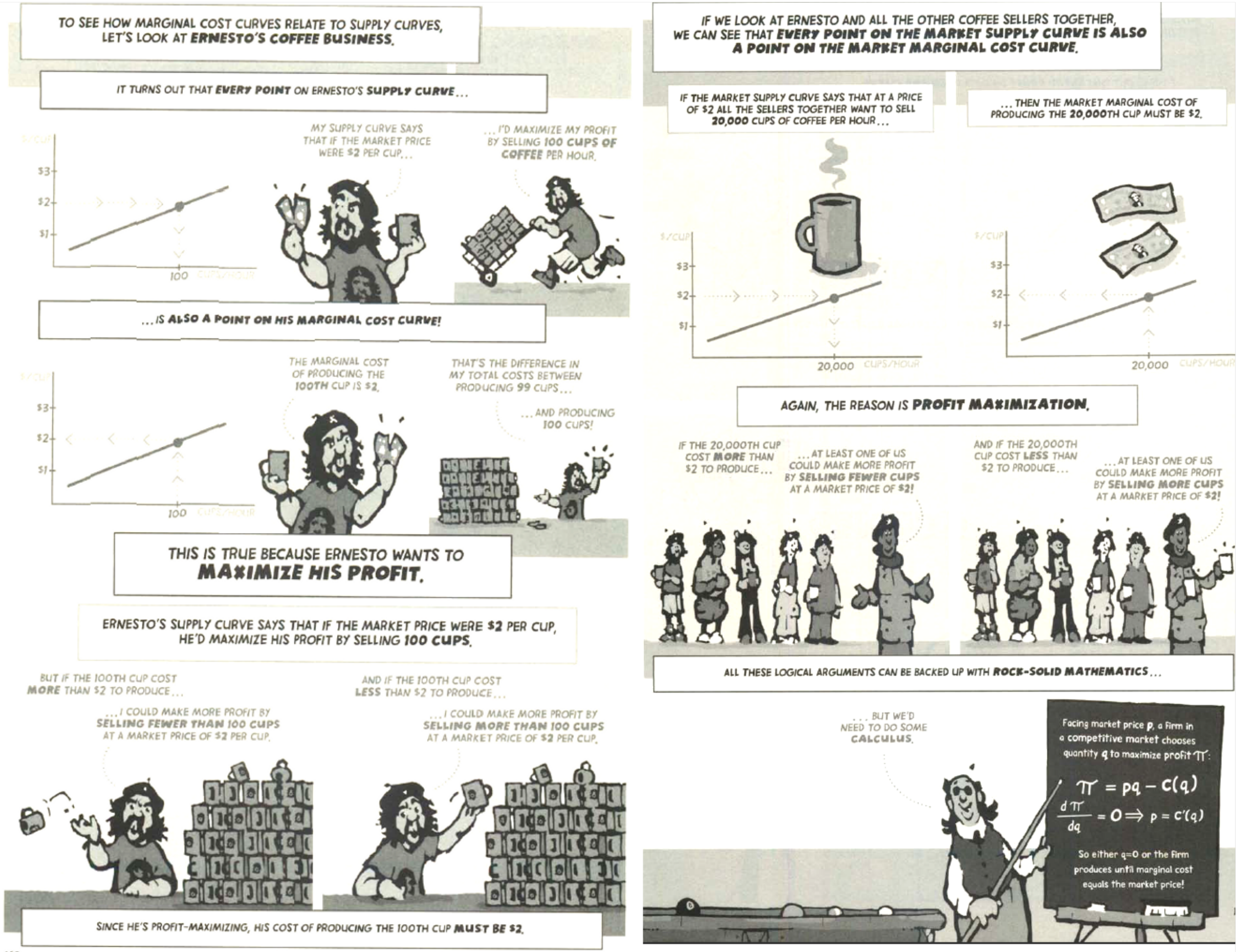
$$\text{MC} - \text{ATC} = 0 \tag{9.20}$$

Now our task of proving that ATC is equal to MC when ATC is at its minimum is easy. Having taken the derivative of ATC in Equation 9.14 to show its minimum, we have worked all the way to Equation 9.20. This last Equation will hold true, i.e. will correspond to a minimum of the ATC curve when we set MC and ATC equal to each other. Hence, when MC is equal to ATC, ATC is at its minimum. The same mathematical steps can be followed to prove the intersection of AVC and MC at the minimum of AVC.

Also in



Cartoon Guide introduction to
Economics I, Microeconomics
by Grady Klein and Yoram Bauman



TO SEE HOW MARGINAL COST CURVES RELATE TO SUPPLY CURVES, LET'S LOOK AT ERNESTO'S COFFEE BUSINESS.
IT TURNS OUT THAT EVERY POINT ON ERNESTO'S SUPPLY CURVE...
$3
$2
$1
100
MY SUPPLY CURVE SAYS THAT IF THE MARKET PRICE WERE $2 PER CUP...
...I'D MAXIMIZE MY PROFIT BY SELLING 100 CUPS OF COFFEE PER HOUR.
...IS ALSO A POINT ON HIS MARGINAL COST CURVE!
$3
$2
$1
100
THE MARGINAL COST OF PRODUCING THE 100TH CUP IS $2.
THAT'S THE DIFFERENCE IN MY TOTAL COSTS BETWEEN PRODUCING 99 CUPS...
...AND PRODUCING 100 CUPS!
THIS IS TRUE BECAUSE ERNESTO WANTS TO MAXIMIZE HIS PROFIT.
ERNESTO'S SUPPLY CURVE SAYS THAT IF THE MARKET PRICE WERE $2 PER CUP, HE'D MAXIMIZE HIS PROFIT BY SELLING 100 CUPS.
BUT IF THE 100TH CUP COST MORE THAN $2 TO PRODUCE...
...I COULD MAKE MORE PROFIT BY SELLING FEWER THAN 100 CUPS AT A MARKET PRICE OF $2 PER CUP.
AND IF THE 100TH CUP COST LESS THAN $2 TO PRODUCE...
...I COULD MAKE MORE PROFIT BY SELLING MORE THAN 100 CUPS AT A MARKET PRICE OF $2 PER CUP.
SINCE HE'S PROFIT-MAXIMIZING, HIS COST OF PRODUCING THE 100TH CUP MUST BE $2.
IF WE LOOK AT ERNESTO AND ALL THE OTHER COFFEE SELLERS TOGETHER, WE CAN SEE THAT EVERY POINT ON THE MARKET SUPPLY CURVE IS ALSO A POINT ON THE MARKET MARGINAL COST CURVE.
IF THE MARKET SUPPLY CURVE SAYS THAT AT A PRICE OF $2 ALL THE SELLERS TOGETHER WANT TO SELL 20,000 CUPS OF COFFEE PER HOUR...
...THEN THE MARKET MARGINAL COST OF PRODUCING THE 20,000TH CUP MUST BE $2.
$/CUP
$3
$2
$1
20,000
CUPS/HOUR
$/CUP
$3
$2
$1
20,000
CUPS/HOUR
AGAIN, THE REASON IS PROFIT MAXIMIZATION.
IF THE 20,000TH CUP COST MORE THAN $2 TO PRODUCE...
...AT LEAST ONE OF US COULD MAKE MORE PROFIT BY SELLING FEWER CUPS AT A MARKET PRICE OF $2!
AND IF THE 20,000TH CUP COST LESS THAN $2 TO PRODUCE...
...AT LEAST ONE OF US COULD MAKE MORE PROFIT BY SELLING MORE CUPS AT A MARKET PRICE OF $2!
ALL THESE LOGICAL ARGUMENTS CAN BE BACKED UP WITH ROCK-SOLID MATHEMATICS...
...BUT WE'D NEED TO DO SOME CALCULUS.
Facing market price p, a firm in a competitive market chooses quantity q to maximize profit π:
$\pi = pq - C(q)$
$\frac{d\pi}{dq} = 0 \Rightarrow p = C'(q)$
So either q=0 or the firm produces until marginal cost equals the market price!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... BUT WE'D NEED TO DO SOME **CALCULUS**.
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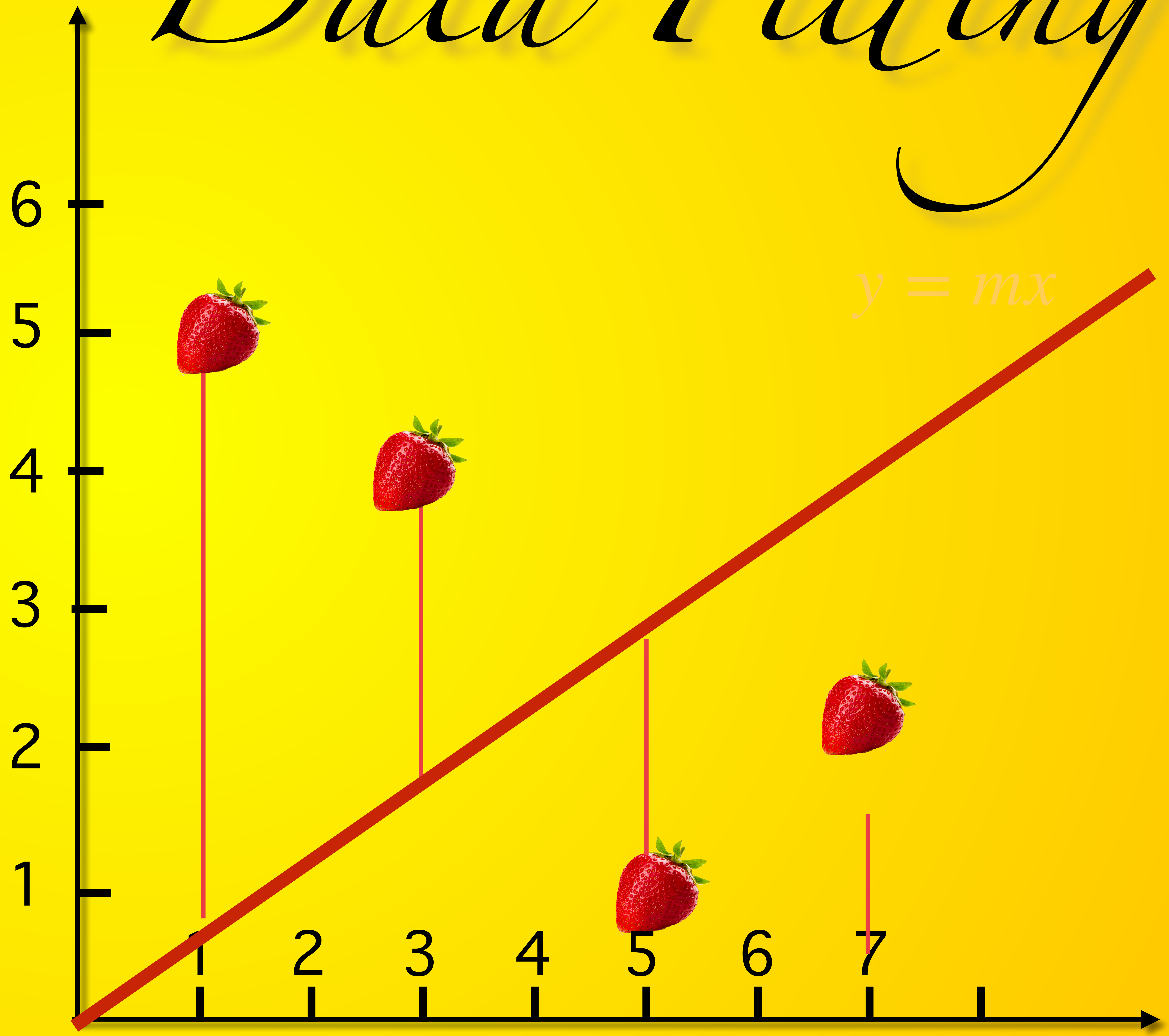
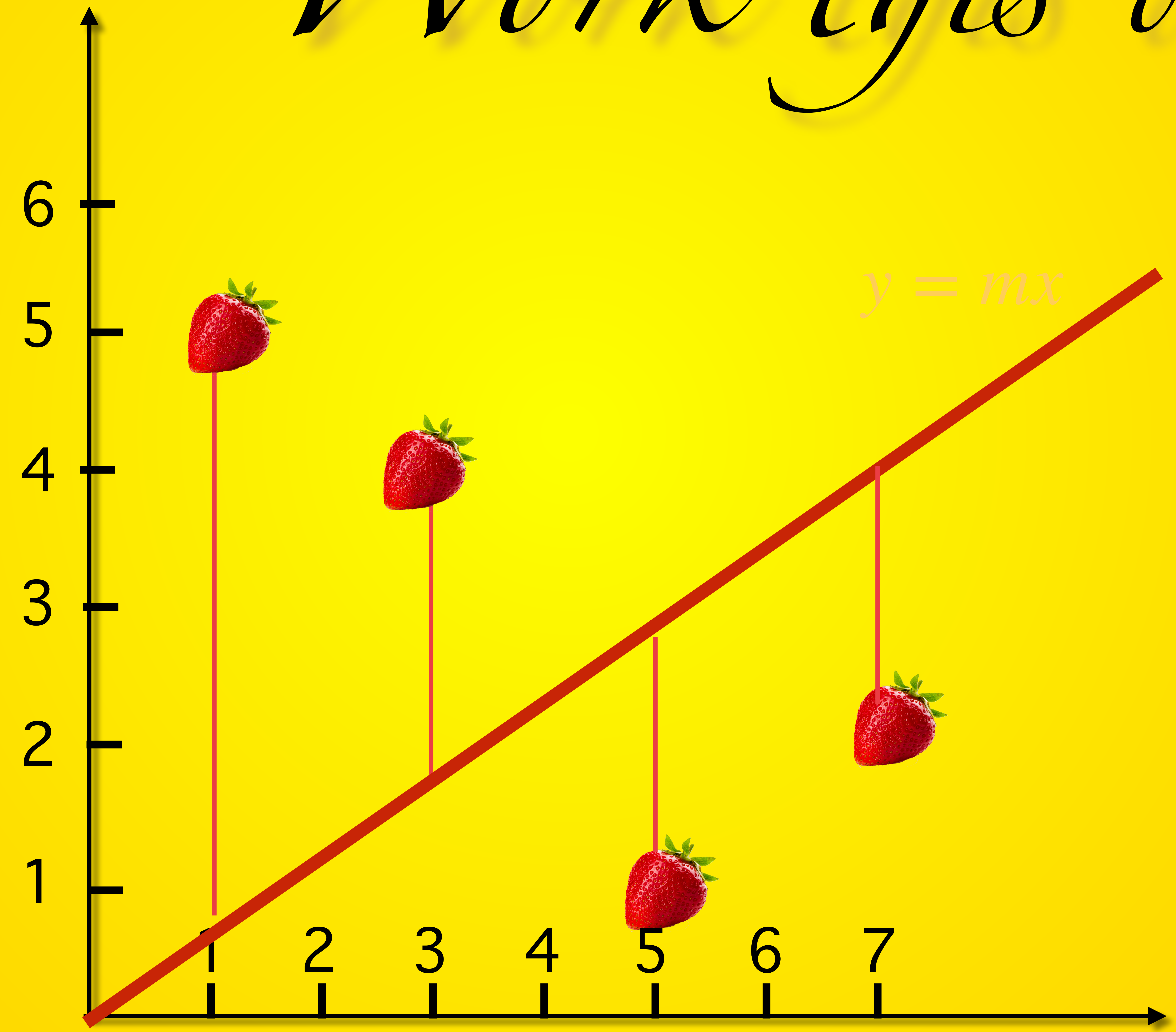
Data Fitting

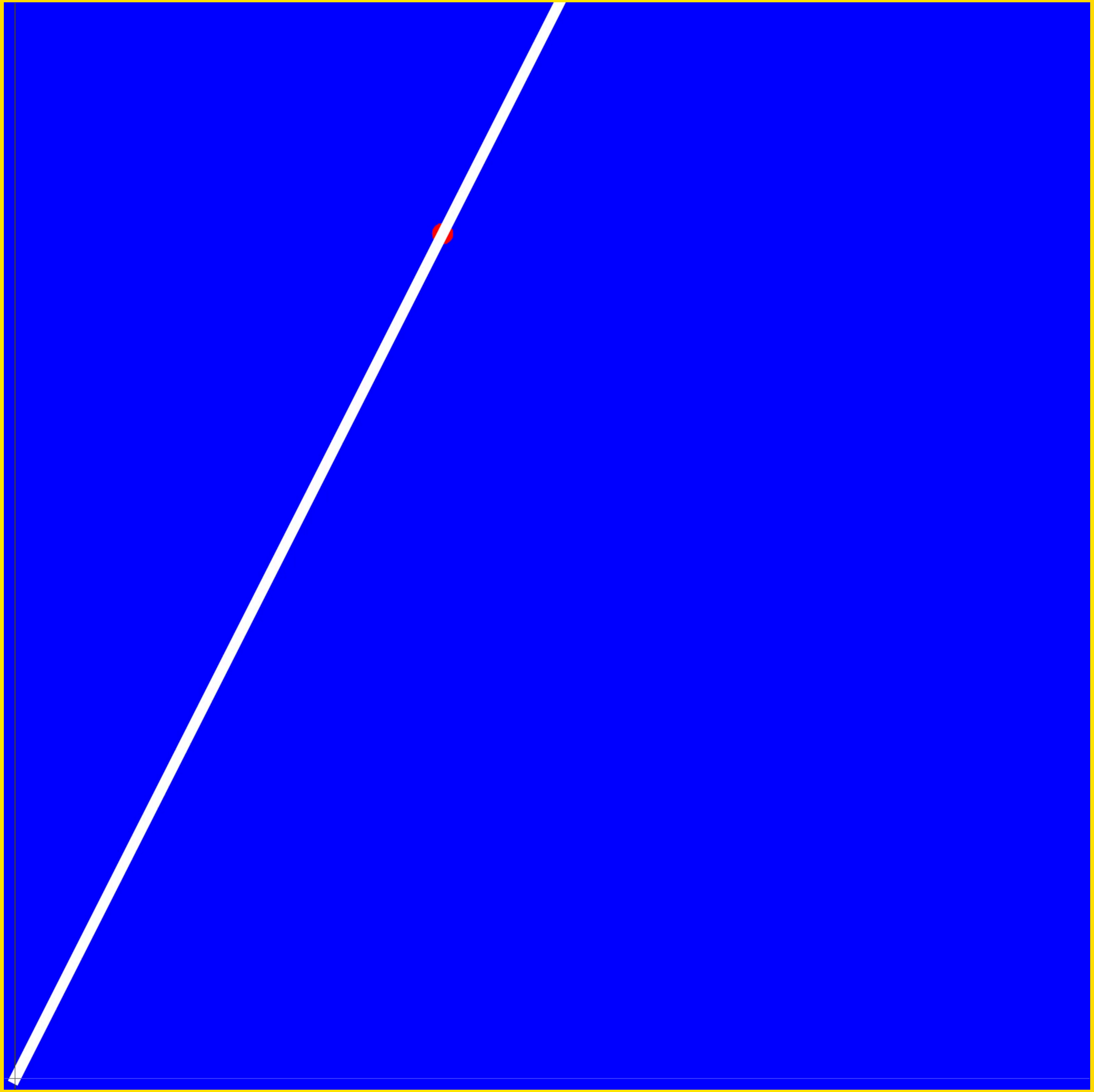
The best fit y=mx minimizes

$$f(m) = \sum (mx_k - y_k)^2$$

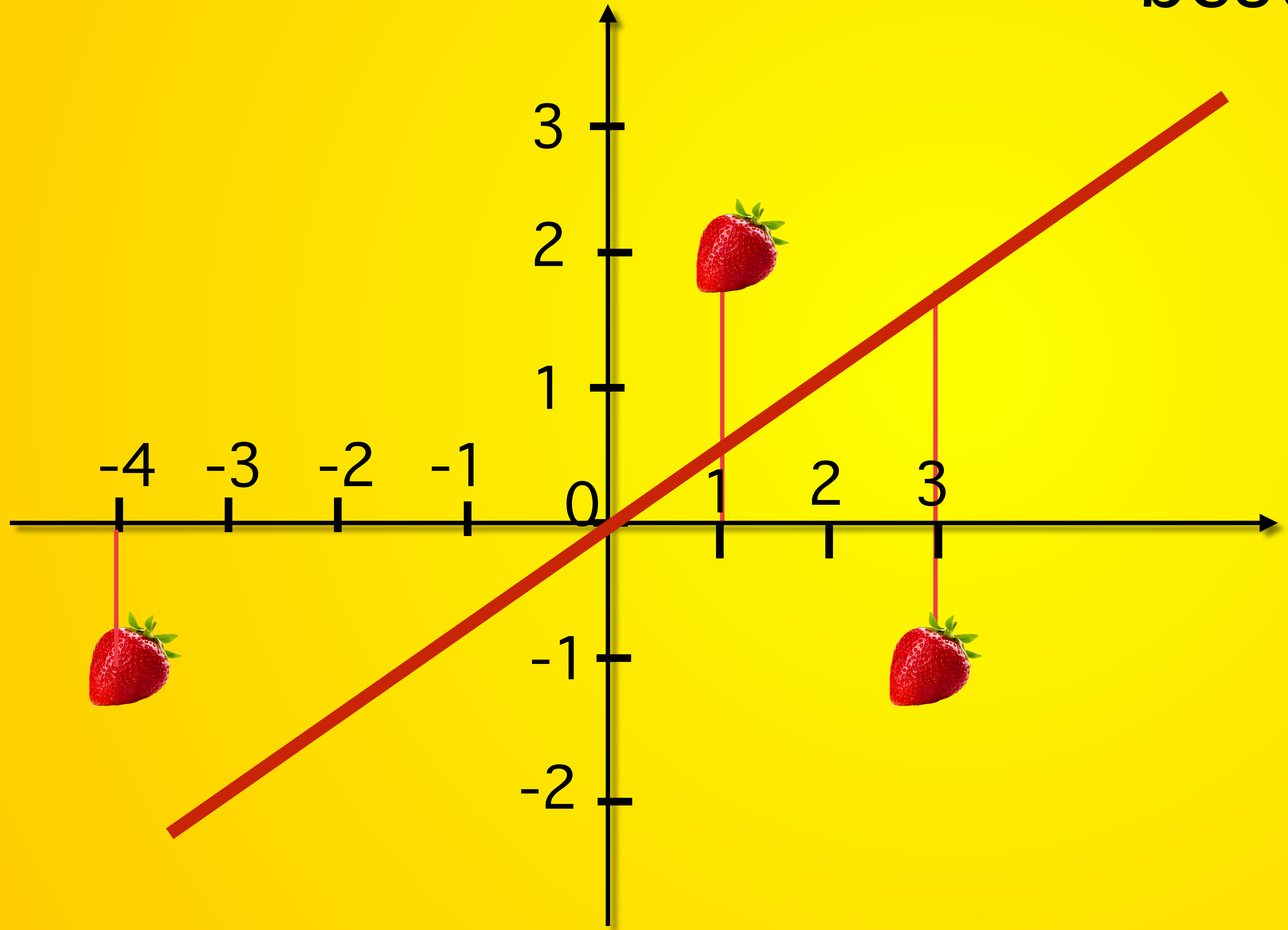


Work this out





JAM



Find the best fit y=mx for

$(-4, -1)$

$(1,2)$

$(3, -1)$

The End