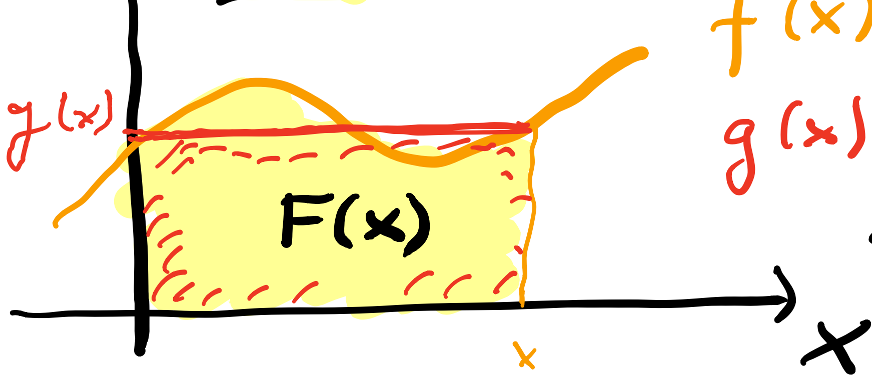
Unit 31



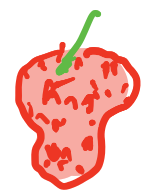
$f(x) = F'(x)$

$g(x) = \frac{F(x)}{x}$

$F = x \cdot g$

Theorem

$$\boxed{f(x) = g(x) \iff g'(x) = 0}$$



$$g = \frac{F}{x}$$

$$g' = \frac{x F' - F \cdot 1}{x^2}$$

$$= \frac{x f - x \cdot g}{x^2}$$

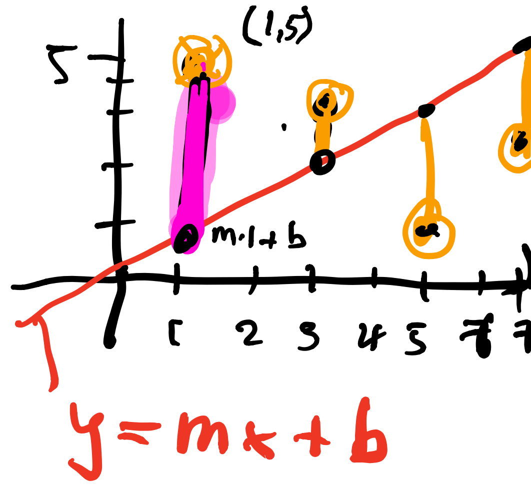
$$= \frac{f - g}{x} = 0$$

"Quite easily done"

QED

"quod erat demonstrandum"

(1, 5)
(3, 4)
(5, 1)
(7, 2)



$$(m\cdot 1 + b - 5)^2 + (m\cdot 3 + b - 4)^2$$
$$+ (m\cdot 5 + b - 1)^2 + (m\cdot 7 + b - 2)^2 = f$$

"Sum of the squares of the distances of the data points to the line"

First center the data

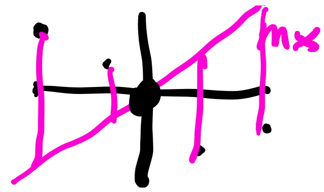
$$\frac{(1,5)+(3,4)+(5,1)+(7,2)}{4} = (4,3)$$

Subtract this

(-3, 2)
(-1, 1)
(1, -2)
(3, -1)

Now we can find the best line

$y = mx$.



$$f(m) = (m\cdot(-3)-2)^2 + (m(-1)-1)^2 + (m\cdot 1+2)^2 + (m3+1)^2$$

$$= 20m^2 + 24m + 10$$

Minimize this!

$$m = -\frac{24}{40} = \boxed{-\frac{3}{5}}$$

$$f' = 40m + 24 = 0$$

$$m = -\frac{24}{40} = -\frac{3}{5}$$

The actual best fit

$$m(x-4) + 3$$

$$-\frac{3}{5}(x-4)+3$$

$$= \boxed{-\frac{3}{5}x+\frac{12}{5}+3}$$

$$\boxed{-0.6x+5.4}$$

Mathematica has this built in!

data

Fit[{{1,5},{3,4},{5,1},{7,2}}, {1,x}, x]

functions to fit
