

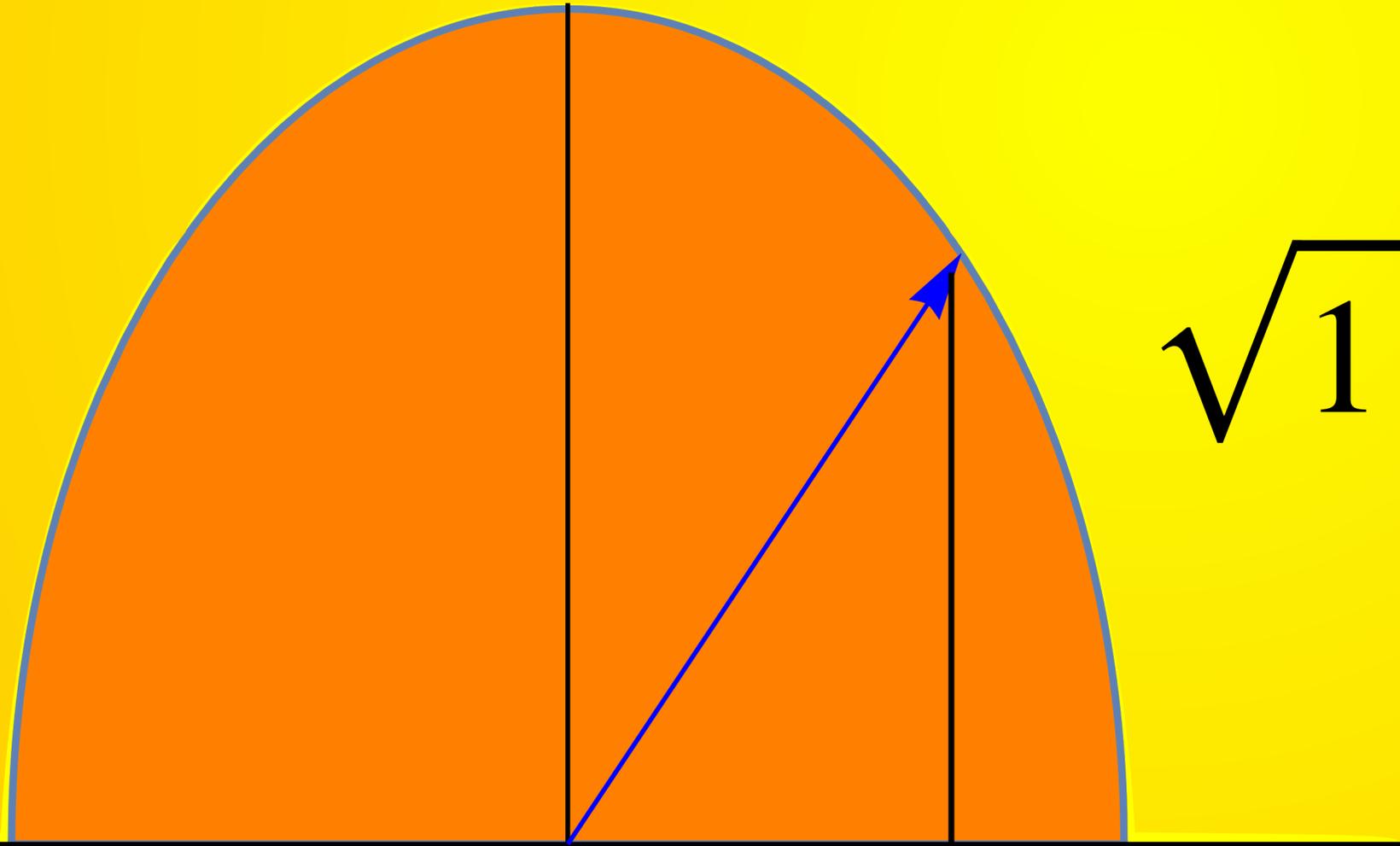
29

Trig Substitutions

$$x = \sin(u)$$

$$dx = \cos(u)du$$

$$\sqrt{1 - x^2} = \cos(u)$$



PLAN

1. Poll

2. A problem

3. Trig substitution

4. Rushmore

5) Integrating rational trigs

6) Examples

Poll

What is
 $\sin^{-1}(x)$?

A

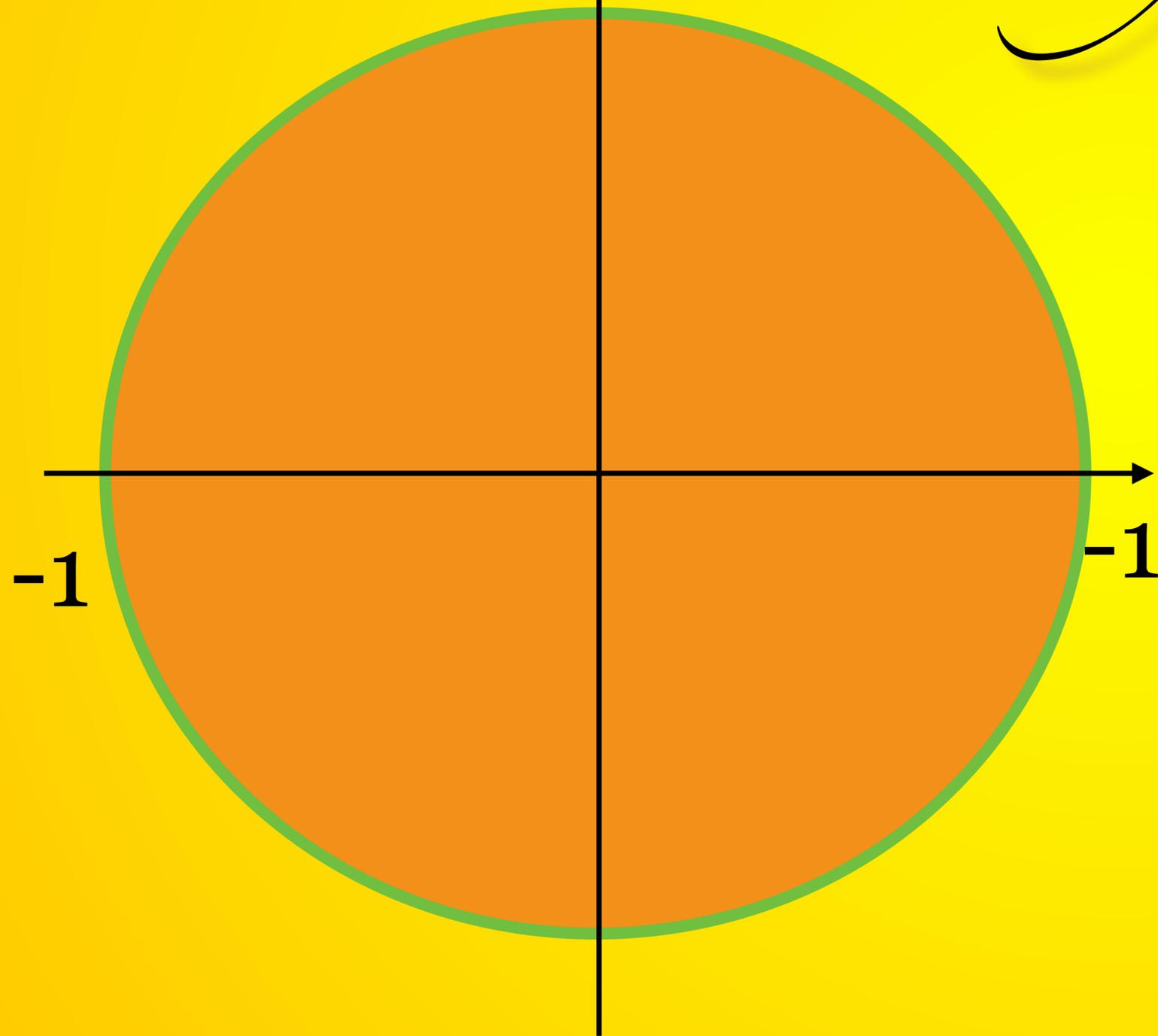
$1/\sin(x)$

B

$\arcsin(x)$

A first problem

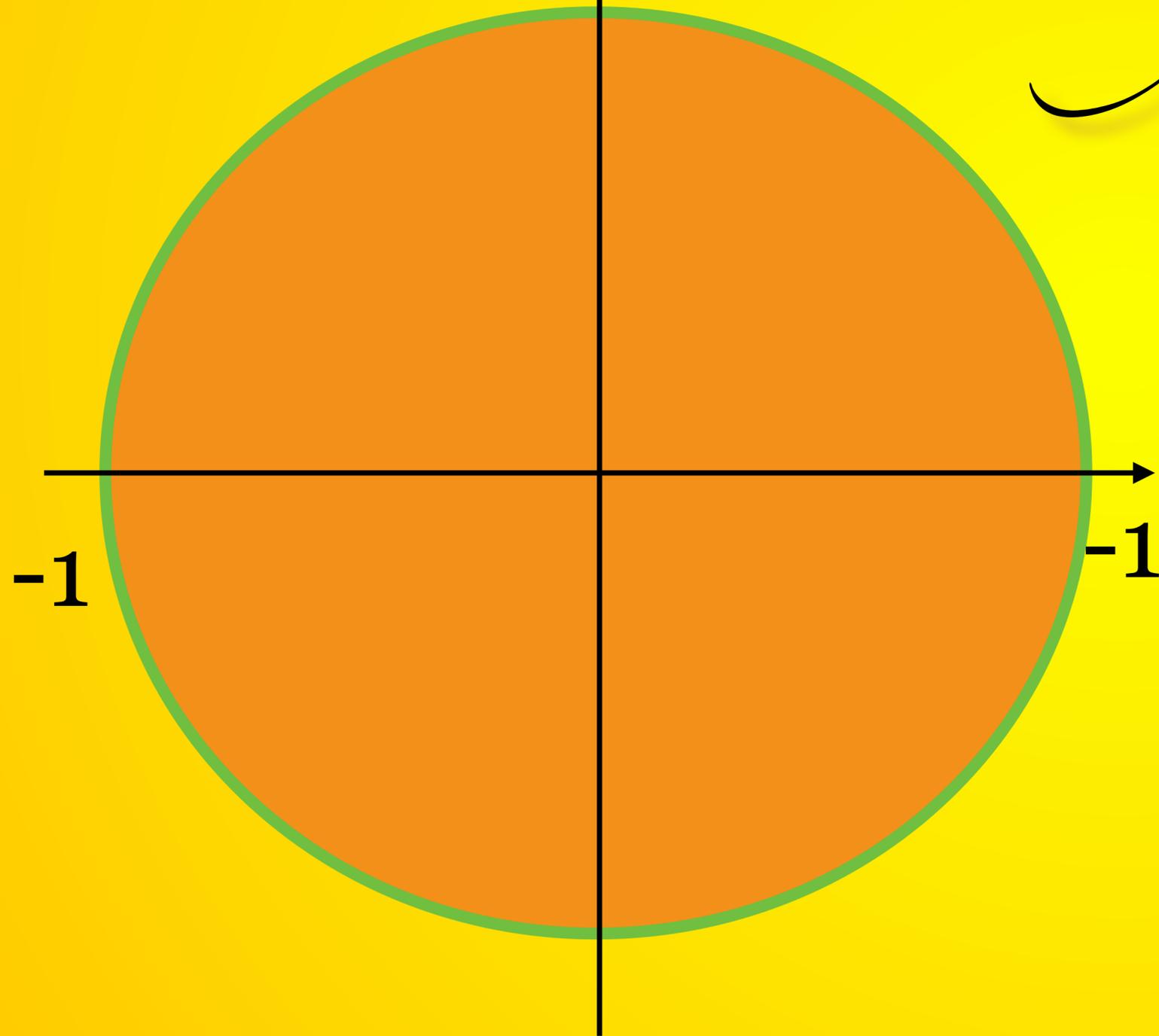
Area of Circle



What was the
area integral
giving the
area of the
circle?

Area of Circle

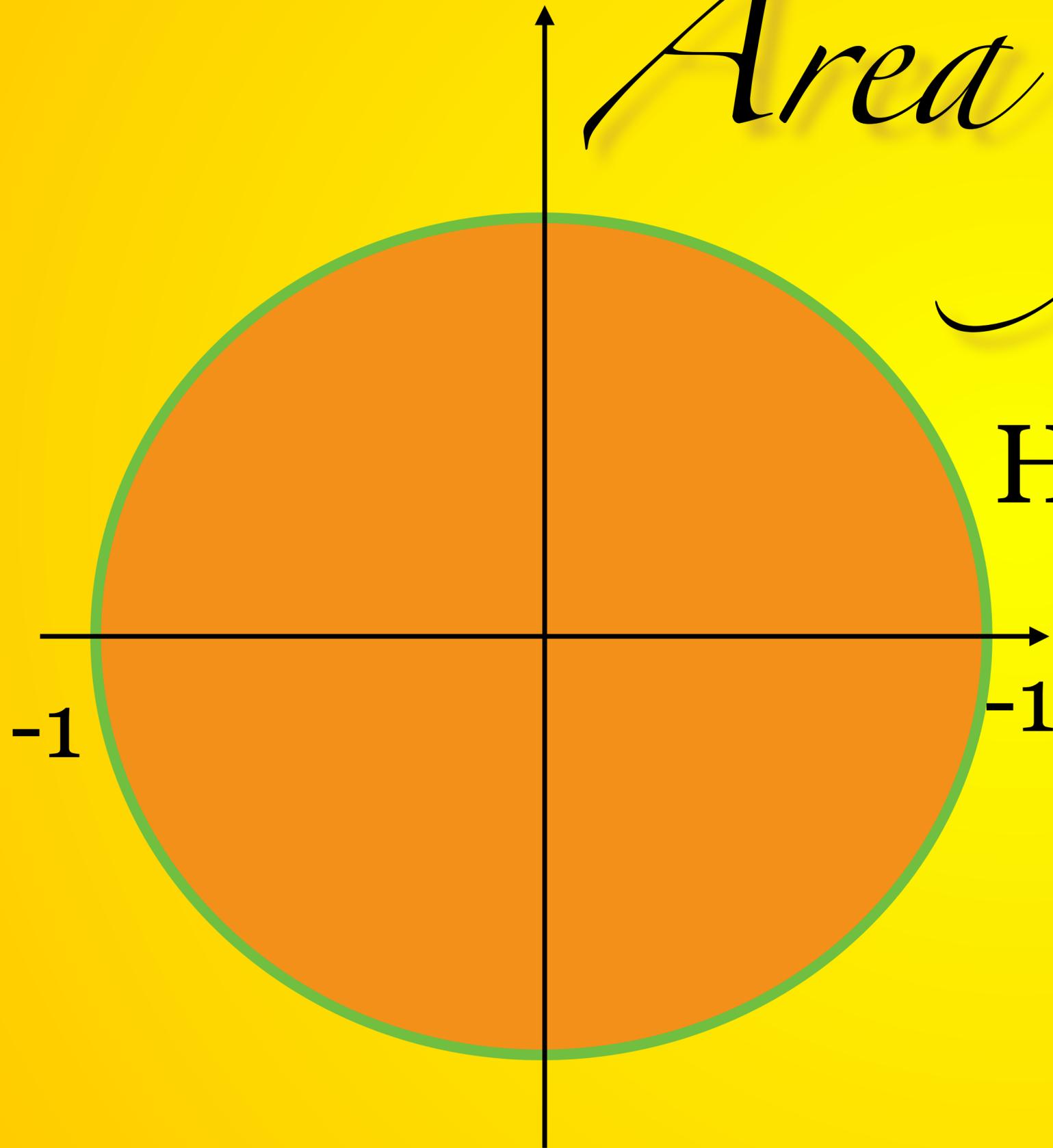
Yes, you got it



$$2 \int_{-1}^1 \sqrt{1-x^2} dx$$

Area of Circle

How do we solve this?



$$2 \int_{-1}^1 \sqrt{1-x^2} dx$$

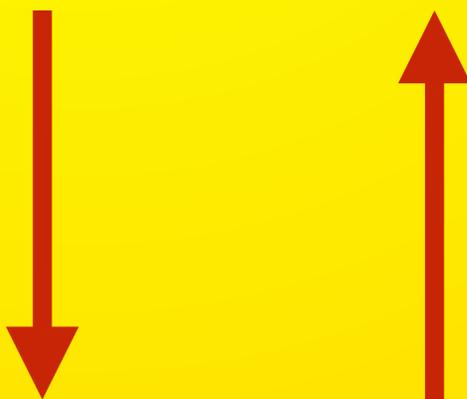
A new integral

How do we solve this integral?

$$\int \sqrt{1 - x^2} \, dx$$

A new integral

1. Attempt: Integration by parts

$$\int \sqrt{1-x^2} \cdot 1 \, dx$$


The diagram shows the integral $\int \sqrt{1-x^2} \cdot 1 \, dx$. A red arrow points downwards from the '1' to the $\sqrt{1-x^2}$ term, and another red arrow points upwards from the $\sqrt{1-x^2}$ term to the '1' term, illustrating the choice of $u = \sqrt{1-x^2}$ and $v = 1$ for integration by parts.

A new integral

2. Attempt: Substitution

$$\int \sqrt{1 - x^2} \, dx =$$

$$u = 1 - x^2$$

$$du = -2x \, dx$$

We need a better idea

Trig Substitution!

Trig Substitution

$$\int \sqrt{1-x^2} \, dx = \int \cos^2(u) \, du$$

$$x = \sin(u)$$
$$dx = \cos(u) du$$

Now solve this
and back substitute

Examples

1

$$\int \frac{1}{\sqrt{1-x^2}} dx$$

2

$$\int (1-x^2)^{3/2} dx$$

Rushmore

EXTRA CREDIT

$$\text{Ellipse} = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

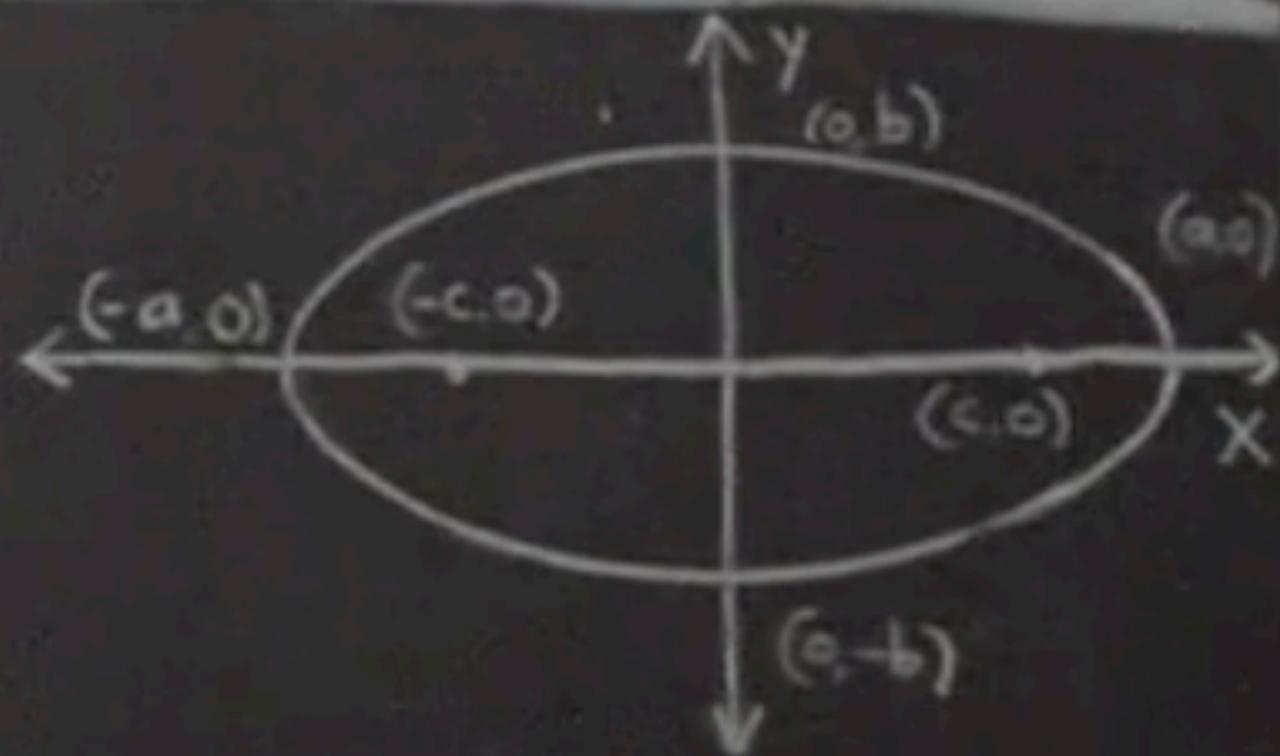
$$e \rightarrow 0 \Rightarrow a \rightarrow b \rightarrow r$$

$$e \rightarrow 0 \Rightarrow c \rightarrow 0$$

$$A_{\odot} = \pi r^2$$

$$\text{TO PROVE: } A_E = \pi ab$$

$$\left(A_E = \int_{-a}^a (y_2 - y_1) dx \right)$$







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A second problem

Examples

What is $\int \frac{1}{\cos(u)} dx$?

Magic

For any rational expression in trig function,
the following substitution works and reduces it to a rational expression.

$$x = \tan\left(\frac{u}{2}\right)$$

$$dx = \frac{2du}{1 + u^2}$$

$$\sin(x) = \frac{2u}{1 + u^2}$$

$$\cos(x) = \frac{1 - u^2}{1 + u^2}$$

Try it:

1 What is

$$\int \frac{1}{\sin(x)} dx$$

?

2 What is

$$\int \frac{\tan^2(x/2)}{\sin(x)} dx$$

?

Use $x = \cos(u)$

Jam

A $\int \sqrt{1 - x^2} \, dx$

Use the magic box

B $\int \frac{\arccos(x)}{\sqrt{1 - x^2}} \, dx$

C $\int \frac{\tan(x/2)}{\sin(x)} \, dx$

The End