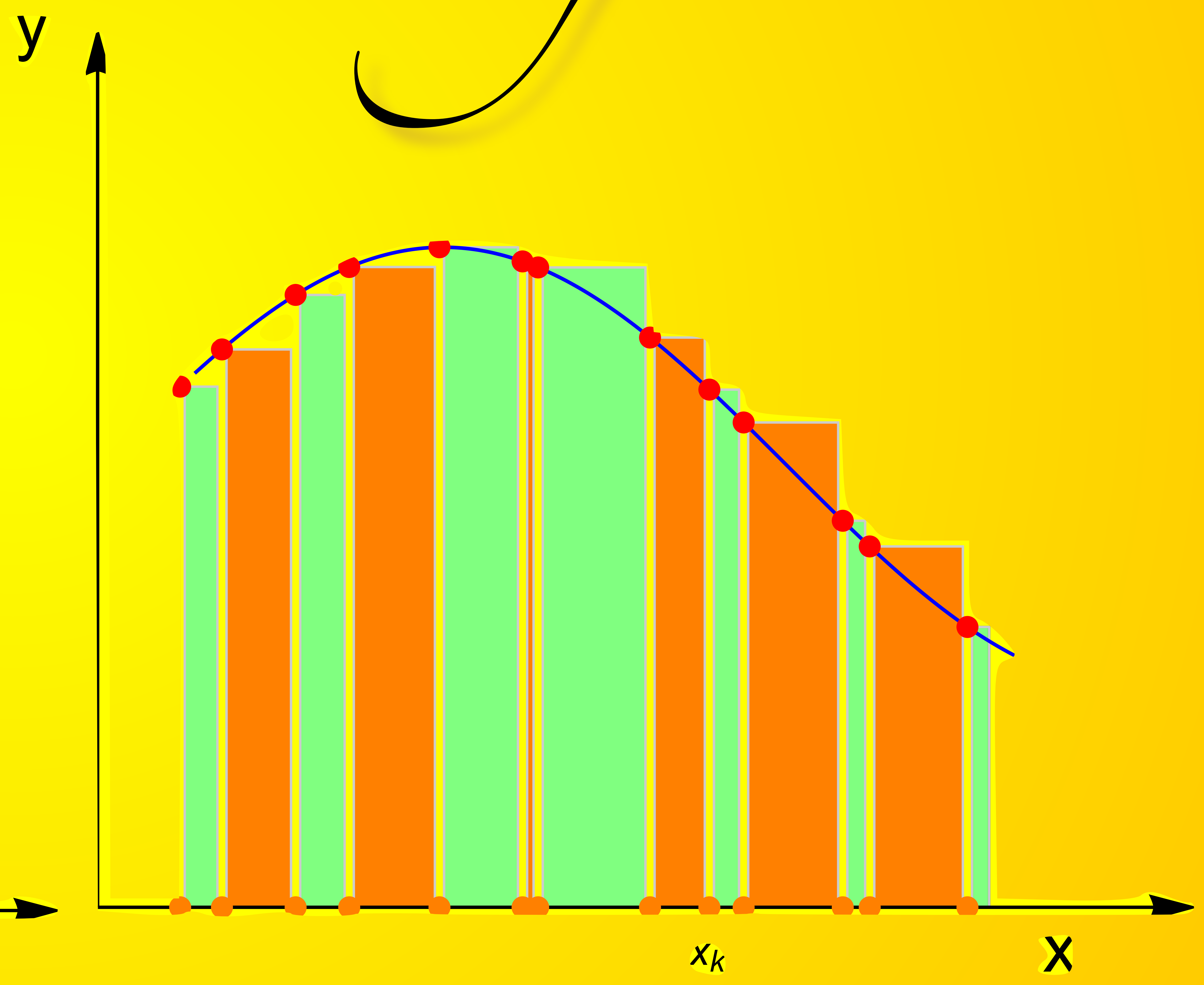
27 Numerical Integration



Numerics Pioneers

Kepler

Riemann

Simpson

PLAN

1. Poll

2. General Riemann Sums

3.Left and Right sums

4. Trapezoid Mathod

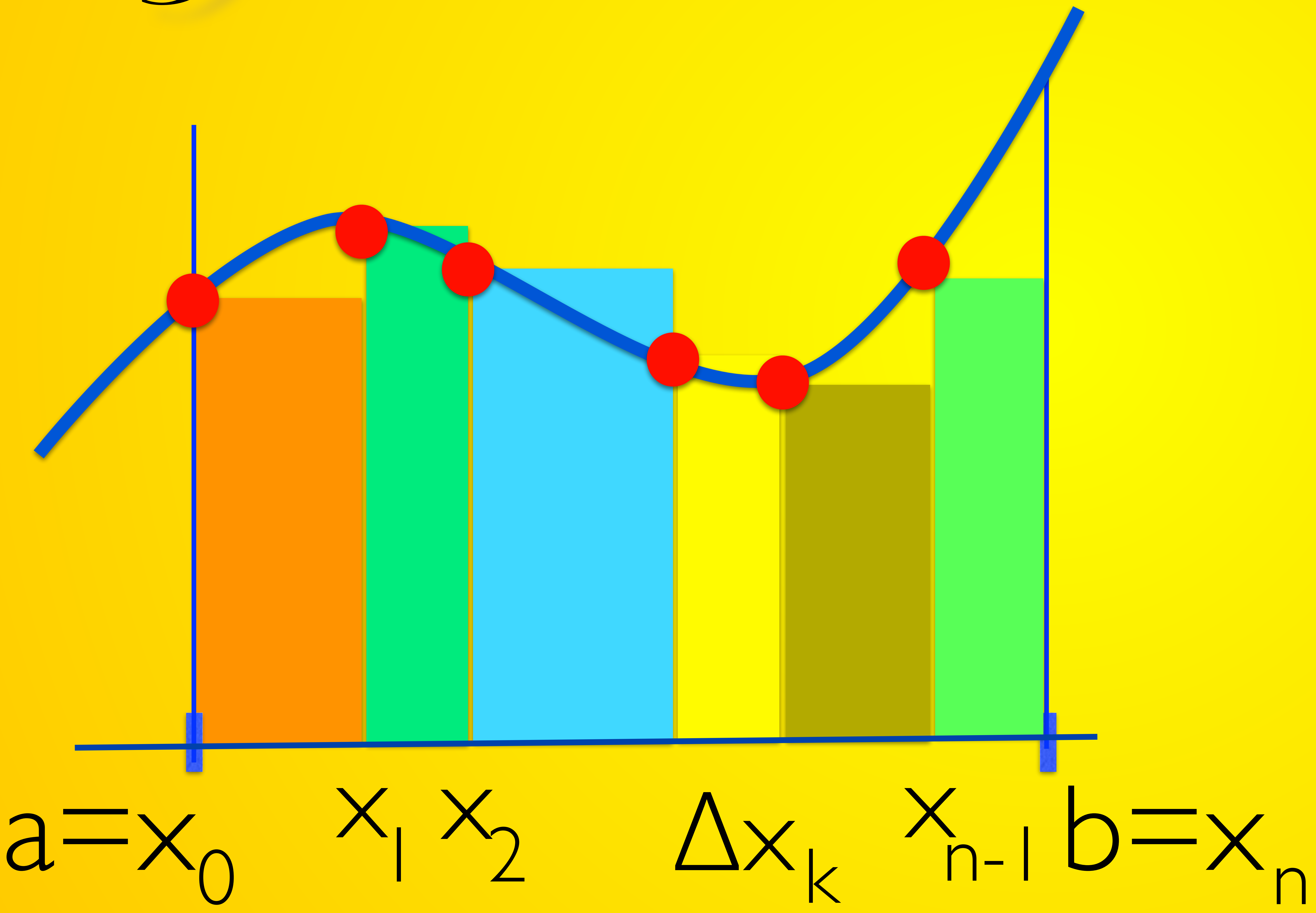
5.Simpson Method

6.Simpson 3/8 Method

7.Monte Carlo Mathod


CASINO ROYALE

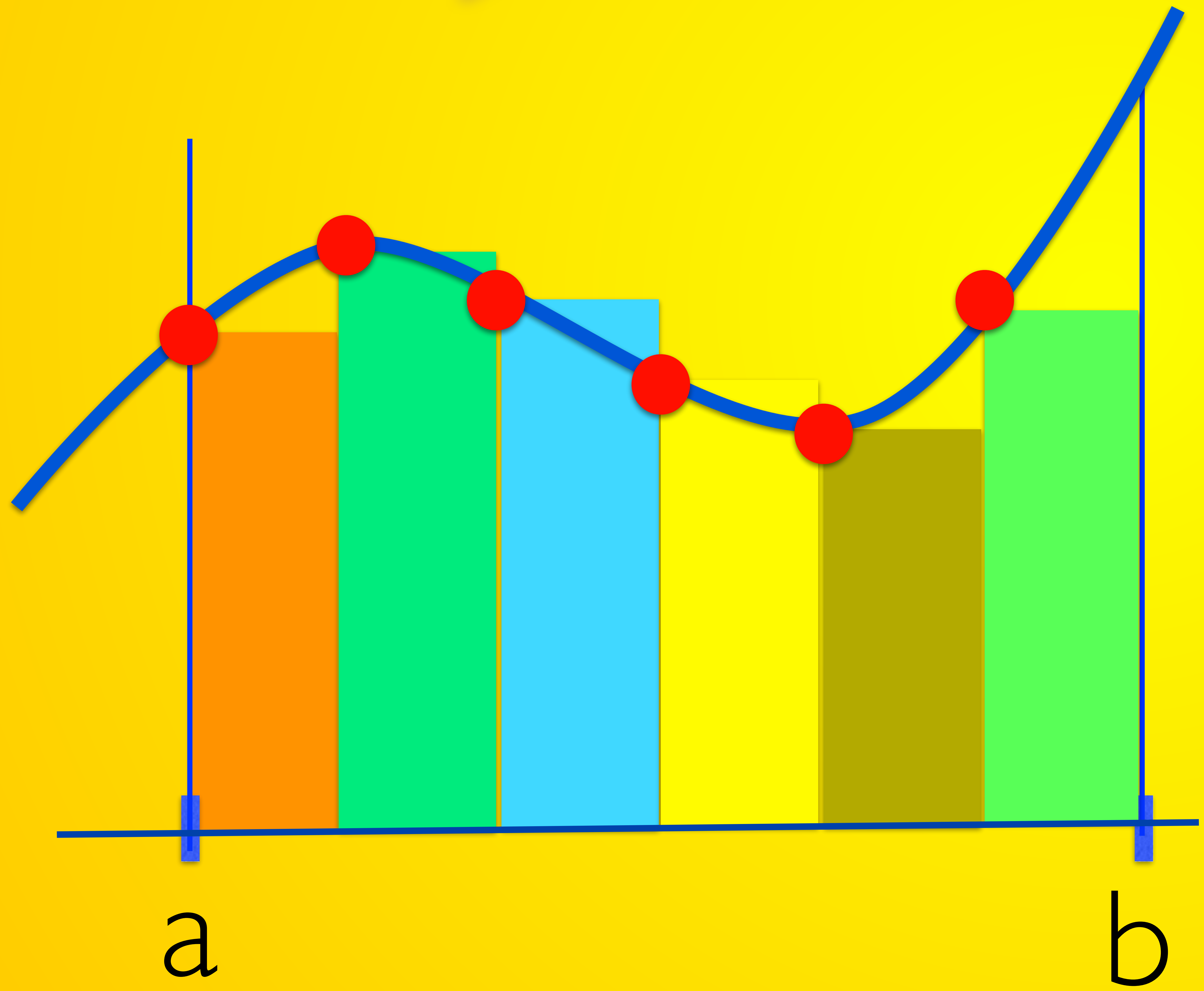
General Riemann Sum



$$\sum_{k=0}^{n-1} f(x_k)\Delta(x_k)$$

$$\int_a^b f(x)\, dx$$

Left Riemann Sum



$$x_k = a + k\frac{b-a}{n}$$

$$L = \sum_{k=0}^{n-1} f(x_k)\Delta(x_k)$$

Poll Movie Trivia

Which game was played in the high stakes play in the bond movie "Casino Royale"?

A Poker

C Blackjack

B Roulette

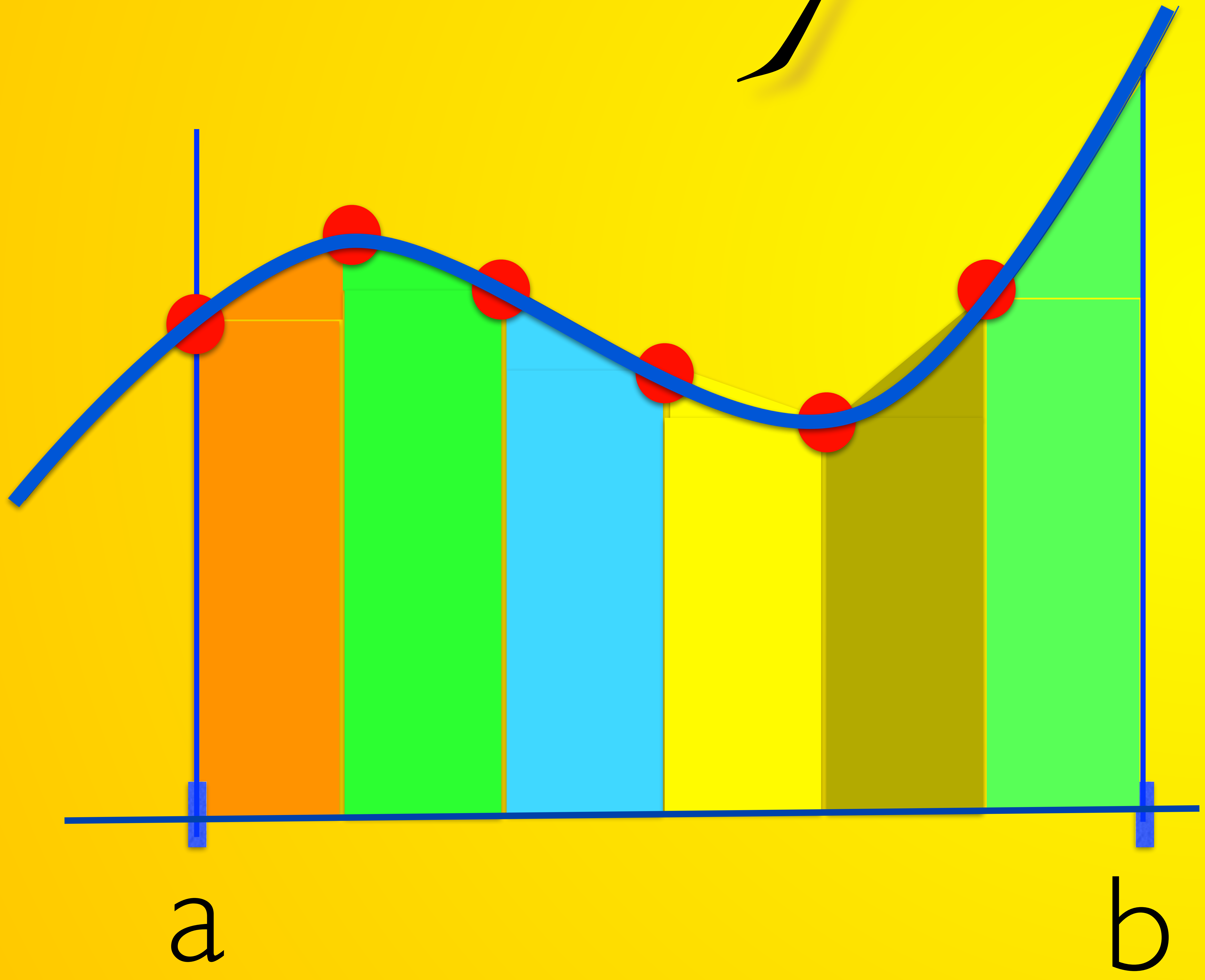
D Backgammon

Right Riemann Sum

$$x_k = a + k\frac{b-a}{n}$$

$$R = \sum_{k=1}^{n} f(x_k)\Delta(x_k)$$

Trapezoid Rule



$$x_k = a + k\frac{b-a}{n}$$

$$\frac{L+R}{2} = \sum_{k=1}^{n} f(x_k)\Delta(x_k)$$

Computation

Lets work together
an example x^4 for all
three cases on [0,1] with n=5.

Experiment

Trapezoid

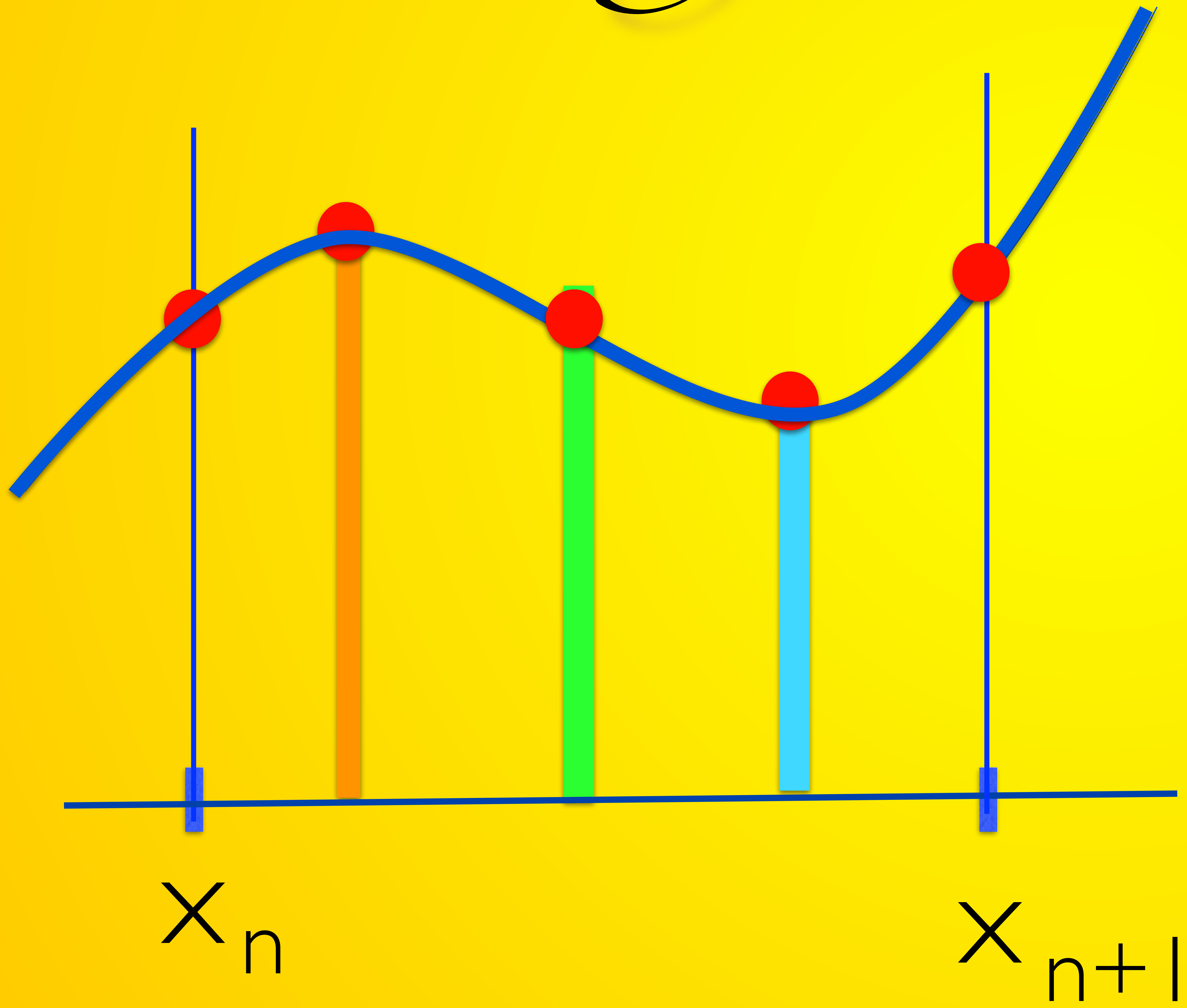
In[1]:=
```
Clear[x]; R = Integrate[ Sin[x], {x, 0, 1}] // N
```
Out[1]= 0.459698

In[2]:=
```
n = 10; A = Sum[ Sin[ k / n], {k, 0, n - 1}] / n  // N
```
Out[2]= 0.417241

In[3]:=
```
n = 10; B = Sum[ Sin[ (k + 1) / n], {k, 0, n - 1}] / n  // N
```
Out[3]= 0.501388

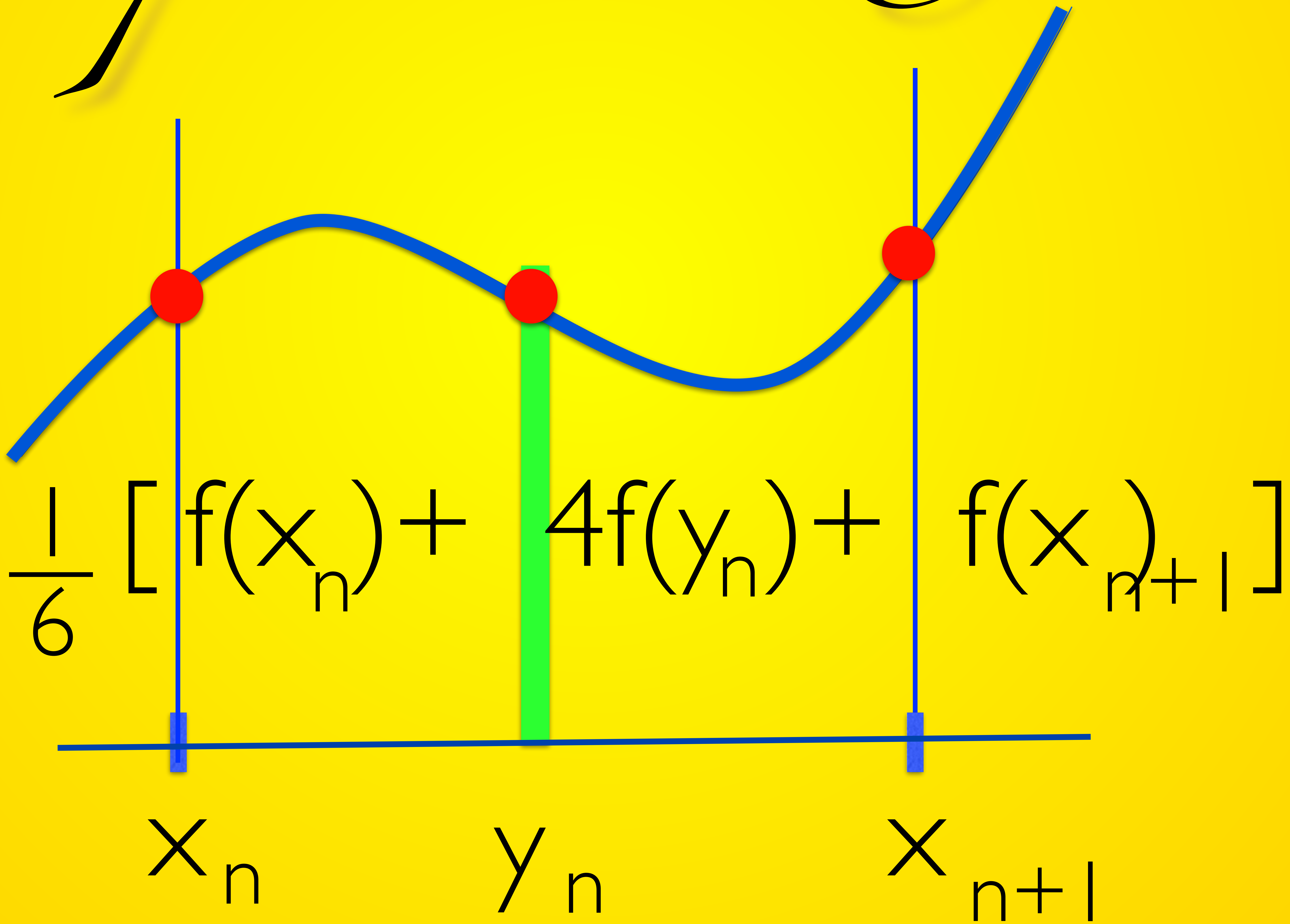
In[4]:=
```
(A + B) / 2  // N
```
Out[4]= 0.459315

Other Methods

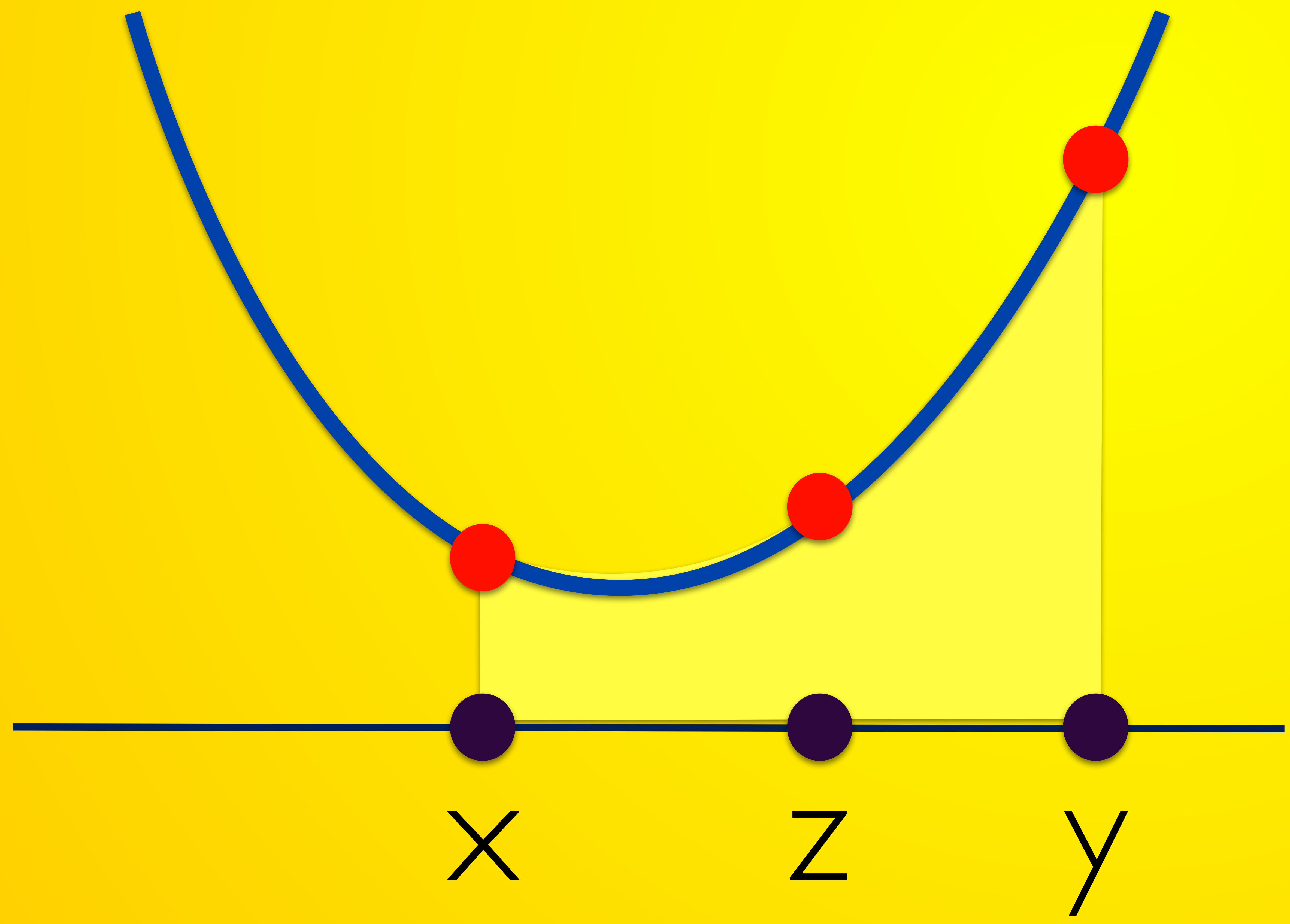


Main Idea:
Use More
Points between
x_n and x_{n+1}

Simpson Method



Example



[f(x)+4f(z)+f(y)]/6 = Area

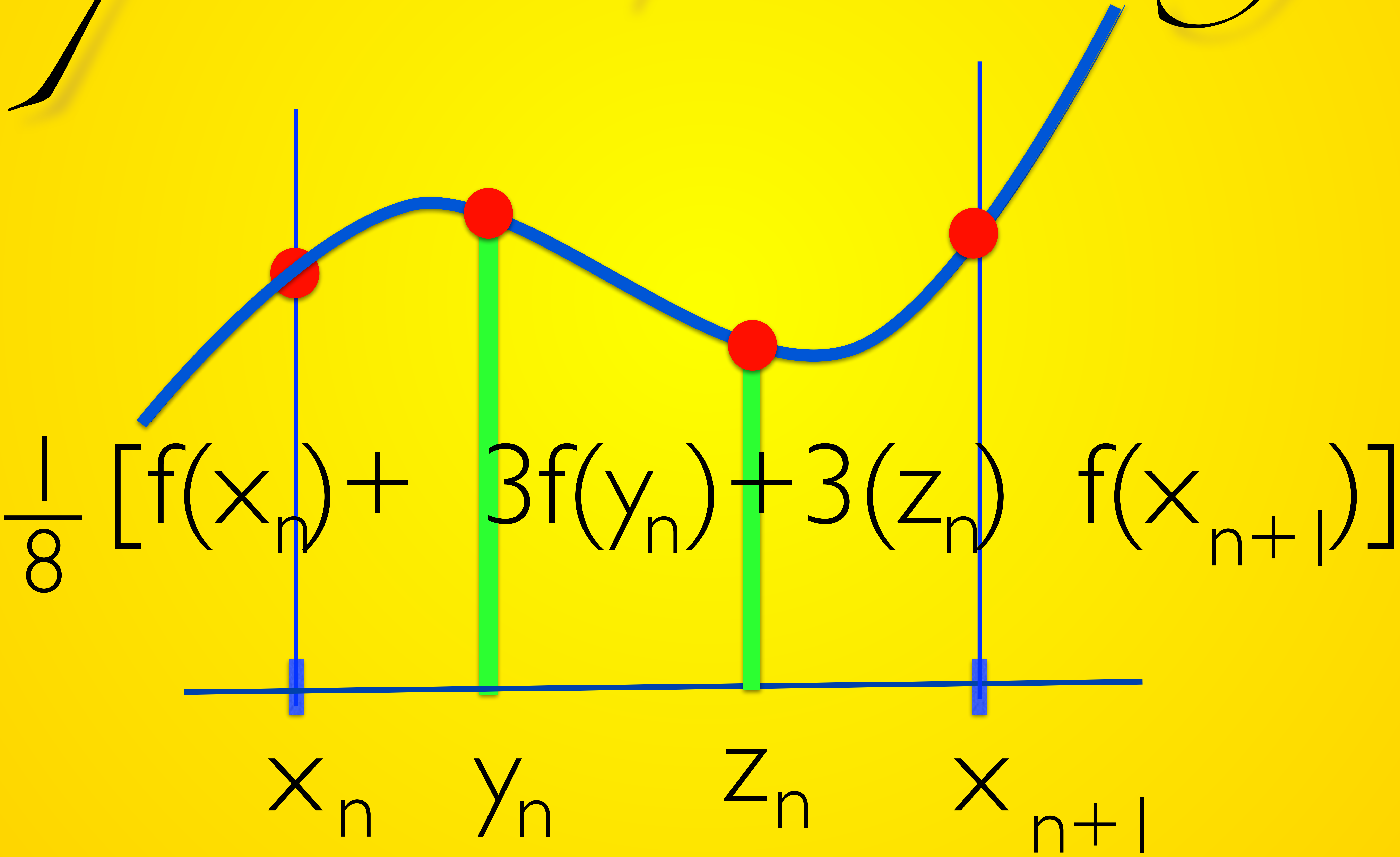
Parabola

Origin

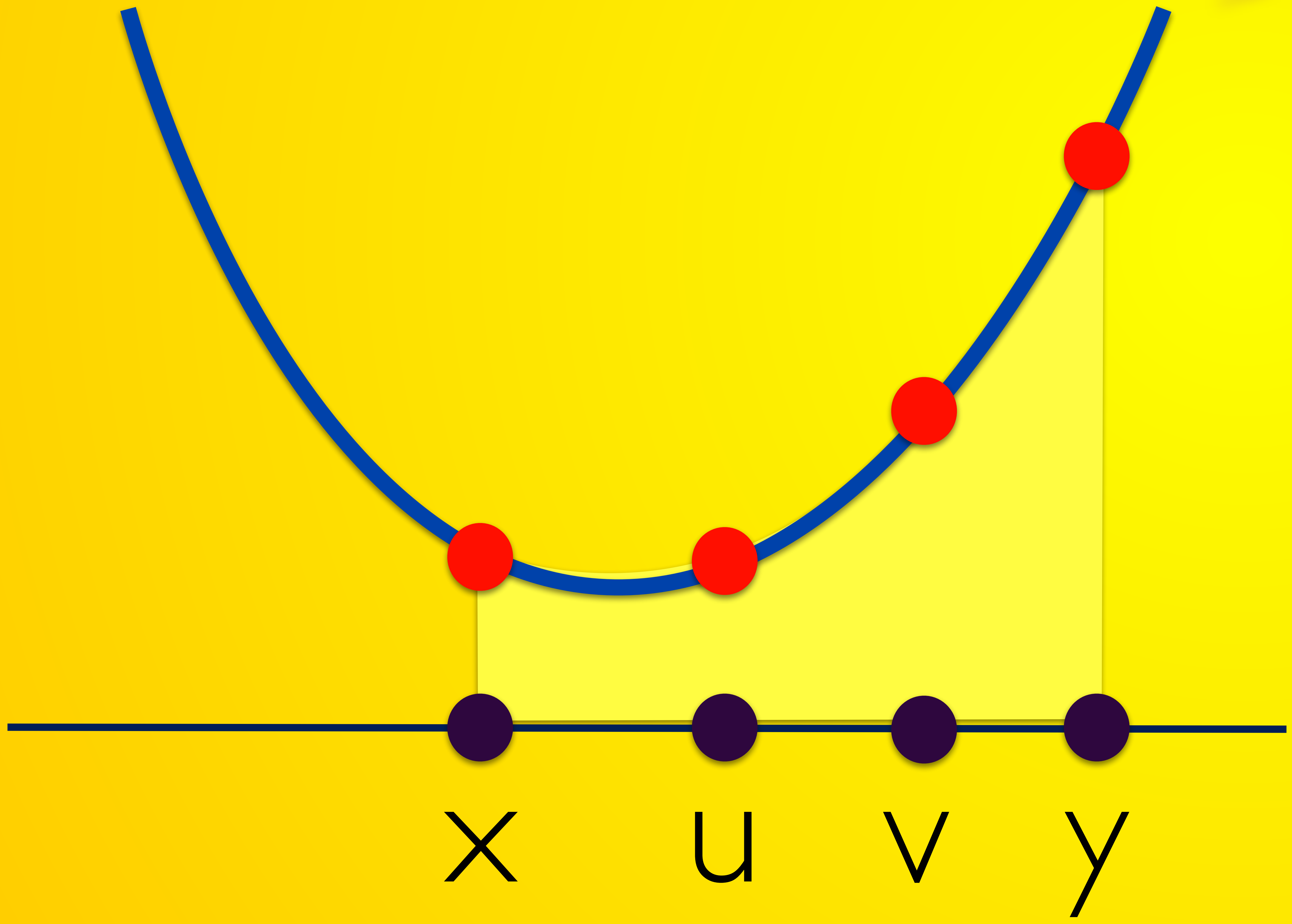
Thomas Simpson
(1710-1761)

Johannes Kepler
(1571-1630)

Simpson 3/8 Method



Example



$[f(x)+3f(u)+3f(v)+f(y)]/8 = \text{Area}$

Parabola

Experiment

Simpson

In[5]:=
```
Clear[x, y]; x[k_] := k / n; y[k_] := (x[k] + x[k + 1]) / 2;
S = Sum[Sin[x[k]] + Sin[x[k + 1]] + 4 Sin[y[k]], {k, 0, n - 1}] / (6 n) //
  N
```

Out[5]=
```
0.459698
```

In[6]:=
```
Clear[x, y, z]; x[k_] := k / n; y[k_] := (2 x[k] + x[k + 1]) / 3;
z[k_] := (x[k] + 2 x[k + 1]) / 3;
T =
 Sum[Sin[x[k]] + Sin[x[k + 1]] + 3 Sin[y[k]] + 3 Sin[z[k]],
    {k, 0, n - 1}] / (8 n) // N
```

Out[7]=
```
0.459698
```

In[8]:=
```
{A - R, B - R, (A + B) / 2 - R, S - R, T - R}
```

Out[8]=

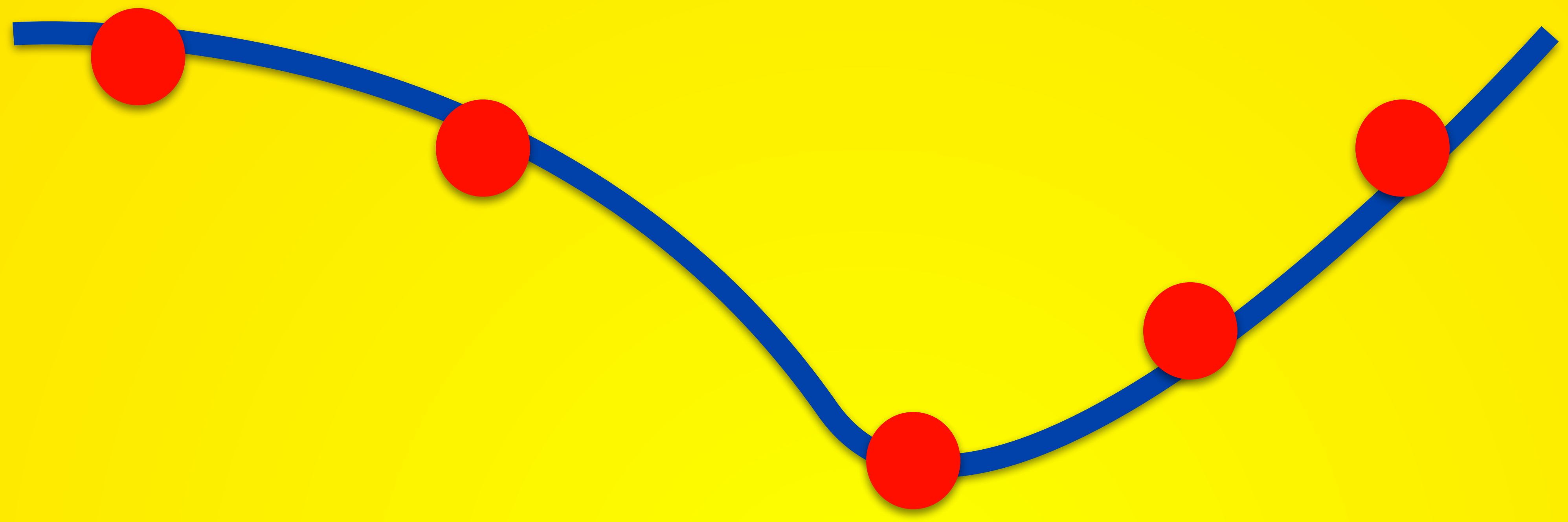
$\{-0.0424567,\ 0.0416904,\ -0.000383145,\ 1.59665\times 10^{-8},\ 7.09598\times 10^{-9}\}$

Computation

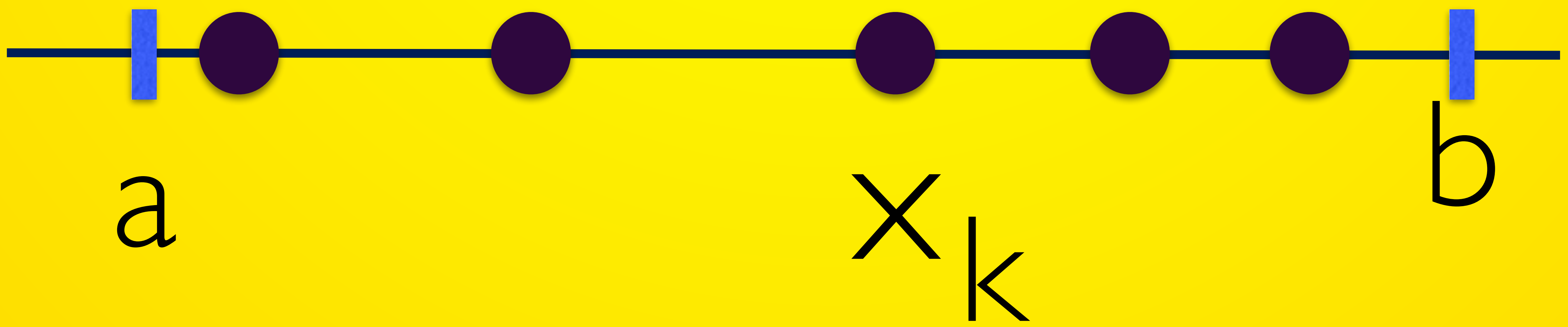
Lets work together
an example x^4 for all
Simpson cases on [0,1] with n=1.

Bond: 1969

Monte Carlo



$$\frac{b-a}{n}\sum_{k=1}^{n} f(x_k)$$



x_k uniformly distributed in [a,b]

Experiment

Monte Carlo

In[16]:=

```
n = 1000;  U := Sum[ Sin[ Random[]], {n}] / n
```

In[17]:=

```
U - R
```

Out[17]=

```
-0.00448761
```

Monte Carlo in 3D

```
R := 2 * Random[] - 1;  n = 10 000;  8 Sum[X = {R, R, R};
    If[X.X < 1, 1, 0], {n}] / n  // N
```

Out[11]=

```
4.1704
```

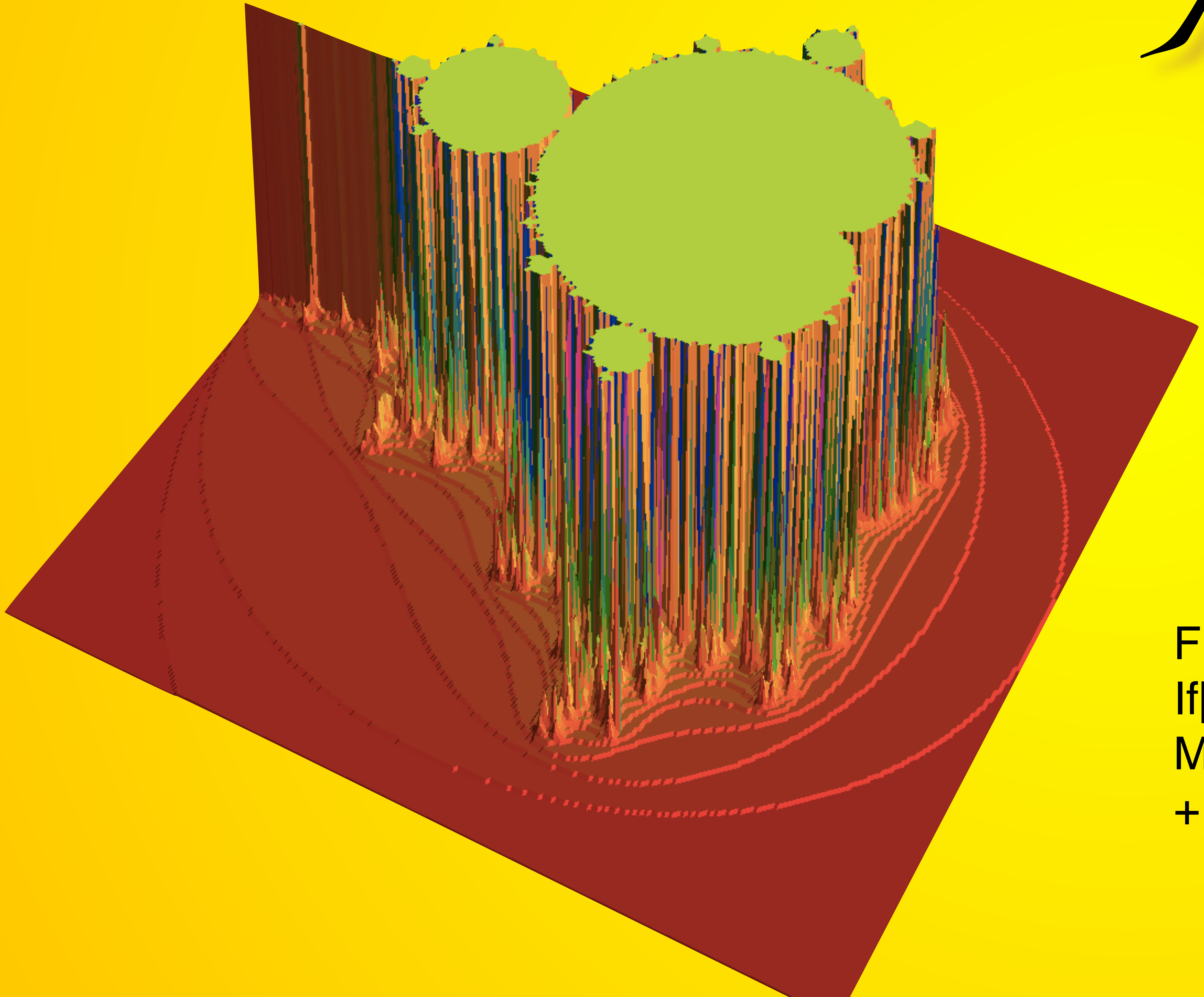
In[12]:=

```
(4 Pi / 3)  // N
```

Out[12]=

```
4.18879
```

An other experiment



```
F[c_] := Module[{z = c}, Do[z = N[z^2 + c];
If[Abs[z] > 3, z = 10], {100}];  If[Abs[z] > 3, 0, 1]];
M = 500; N[9*Sum[F[-2.5 + 3 Random[] + I (-1.5
+ 3 Random[])], {M^2}]/M^2]
```

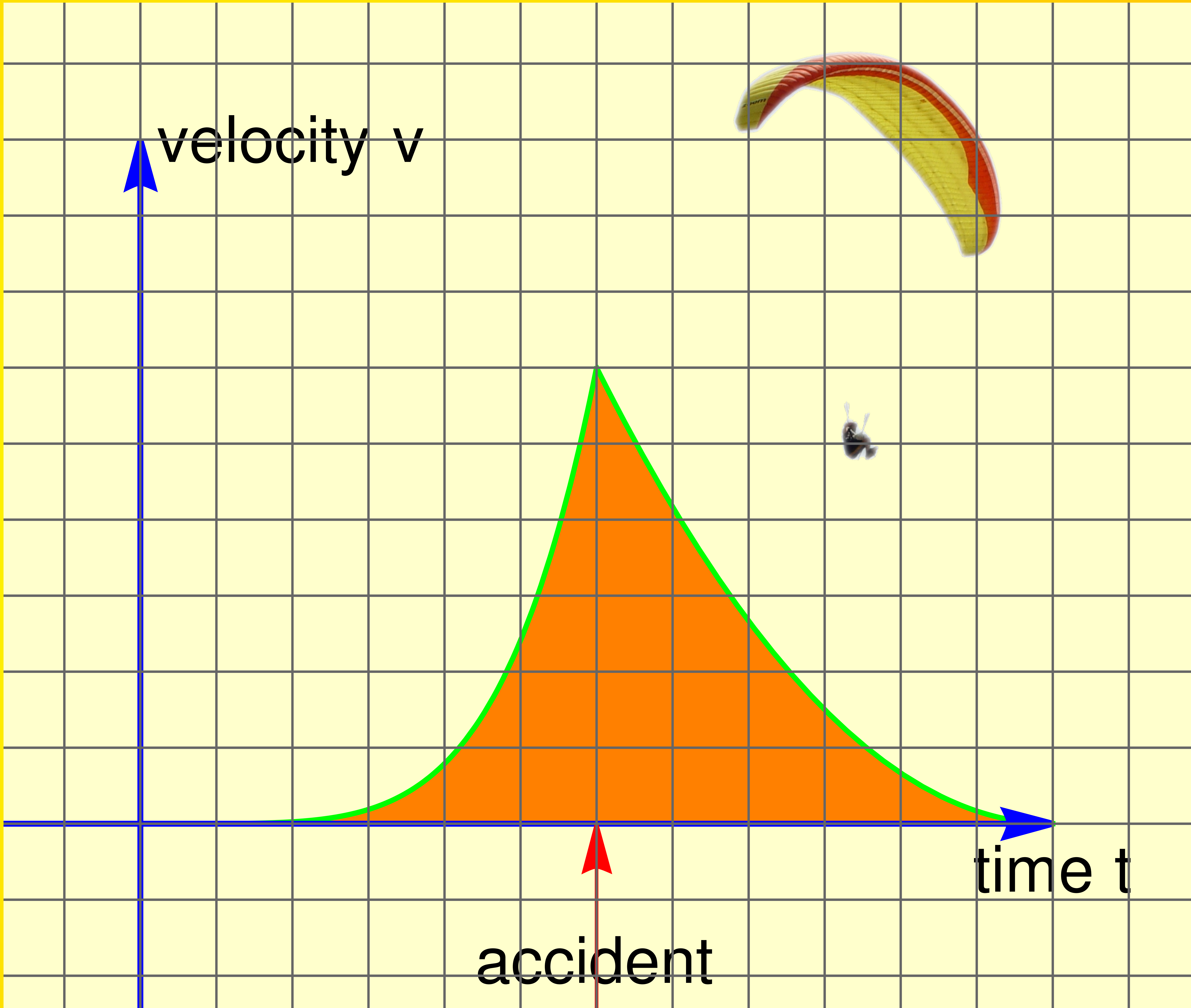

Maths Town: A Trip to Infinity - Mandelbrot Fractal Zoom


W
30
33
250
240
230
220
-60
-65
-60
-70

JAM

a paraglider flies for 12 seconds. At t=6, the wing folds over and the pilot has to land. Estimate the height H at time t=12

$$H = \int_0^{12} v(t)\, dt$$



The End