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Partial Fractions

$$\frac{1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

Hardware upgrades


New rig
Mac mini
Linux
zoom
Please chose
MUSES
DELL
POINCARE'S PRIZE
GEOMETRY

Today: Algebra

Igor Shafarevich once wrote [10]:

In the school mathematical education, algebra has the role of Cinderella and geometry the role of the Beloved Daughter. People would say "I have not chosen mathematics to be my profession but I will remember forever all the beauty of logical construction of geometry" but never anything similar concerning algebra.

Picture source: "Cinderella" by Hans Anker, 1920

Shafarevich then succeeds convincing the reader that algebra contains subdivisions of advanced mathematics like abstract algebra, number theory, combinatorics or even probability theory. And points out in the rest of the article that there is a lot of beauty in the subject.

Exam reminder

April

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

4:◐ 11:● 20:◑ 26:○

Second hourly

Our second takes place on Friday, April 9, 2021. here are practice exams:

- Practice A and Solutions A
- Practice B and Solutions B
- Practice C and Solutions C
- Practice D and Solutions D
- Practice E and Solutions E

practice exams

Problem 1) TF questions (20 points) No justifications are needed.

1)	T	F	The method of partial fractions allows to find anti-derivates of functions like $f(x) = 1/((x-77)(x+78))$
2)	T	F	The method of substitution is based on the chain rule.
3)	T	F	$\int x^3 \, dx = 3x^2 + C$.
4)	T	F	The integral $\int_0^1 f(x) \, dx$ can be approximated by Riemann sums.
5)	T	F	If $f(x) = 1$ everywhere, then $\int_a^b f(x) \, dx$ is the length of the interval $[a, b]$.
6)	T	F	If f is continuous, then $\int_a^b -f(x) \, dx = -\int_a^b f(x) \, dx$.
7)	T	F	If f is a probability density function, then $f(b) - f(a)$ is the probability that the data are in the interval $[a, b]$.
8)	T	F	One can find the anti-derivative of $\sin(4x)\cos(17x)$ using integration by parts.
9)	T	F	The fundamental theorem of calculus implies $\int_a^b f'(x) \, dx = f(b) - f(a)$ if f' is a continuous function.
10)	T	F	If $f(x) = 1/\sqrt{x}$, then the improper integral $\int_0^1 f(x) \, dx$ exists and gives a positive finite area.
11)	T	F	The family $f_c(x) = x^2 + c$ experiences a catastrophe at $c = 0$.
12)	T	F	The anti derivative of $\log(5x)$ is $5x\log(5x) - 5x + C$.
13)	T	F	The volume of a cone of base radius 2 and height 2 is given by the integral $\int_0^2 \pi x^2 \, dx$.
14)	T	F	The volume of a sphere can be derived from the volume of the cone and cylinder.
15)	T	F	If $\lim_{x \to \infty} f(x) = 0$, then $\int_1^\infty f(x) \, dx$ is finite.
16)	T	F	An integral is called improper if it is a convergent indefinite integral.
17)	T	F	The anti-derivative of $\cot(x)$ is $1/(1 + x^2) + C$.
18)	T	F	Tic-Tac-Toe integration appears in the TV Series “Queen’s Gambit”.
19)	T	F	Gabriel’s trumpet is a solid with infinite volume.
20)	T	F	The function $f(x) = e^x$ is called the exponential distribution.
21)	T	F	A PDF is the anti-derivative of the CDF.

Problem 2) Theorems (10 points) No justifications needed.

Fill in the missing part into the empty box to make a true statement.

a) $\frac{d}{dx} \int_1^x f(t) \, dt =$ [ ] by the **fundamental theorem of calculus**.

b) $\int_1^x f'(t) \, dt =$ [ ] by the **fundamental theorem of calculus**.

c) A **probability distribution** is a piece-wise continuous function which is non-negative and satisfies the property [ ].

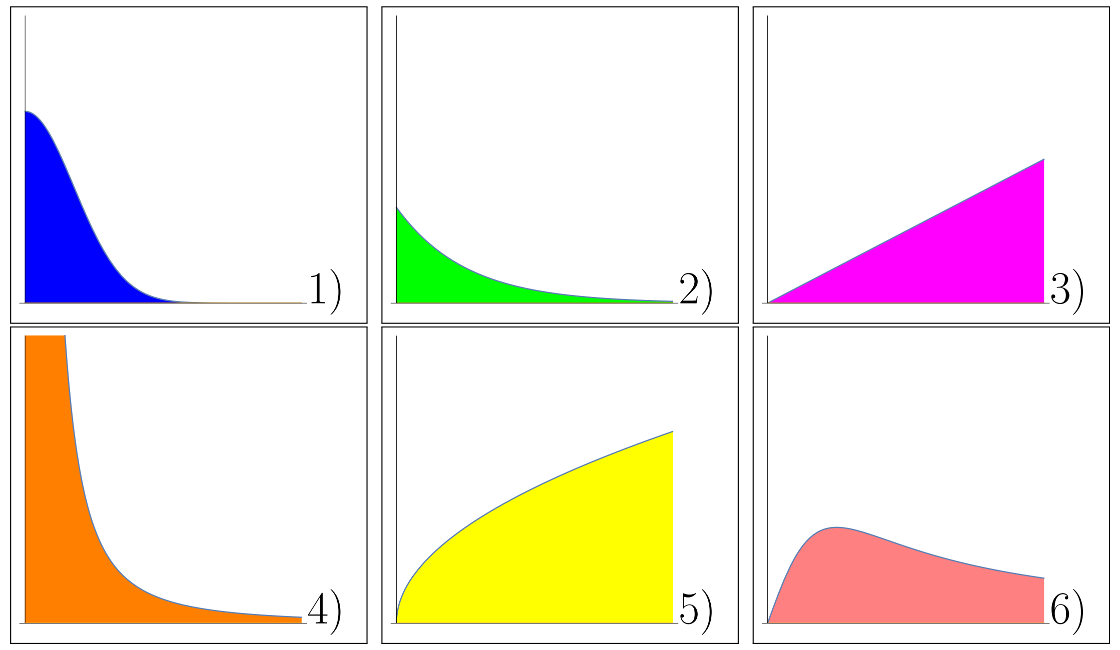
d) The **improper integral** $\int_1^\infty \frac{1}{x^p} \, dx$ converges, if p satisfies the property [ ].

e) Assume $f_c(x)$ is a **family of functions** such that for $c < 0$, there is no minimum and for $c > 0$ there is one minimum, then c is called a [ ].

Problem 3) Matching problem (10 points)

Match the following integrals with parts of the regions and indicate whether the integral represents a finite area.

Integral	Fill in 1-6	Convergent?
$\int_0^\infty x/2 \, dx$		
$\int_0^\infty \frac{1}{x^2} \, dx$		
$\int_0^\infty \sqrt{x} \, dx$		
$\int_0^\infty e^{-x} \, dx$		
$\int_0^\infty e^{-x^2} \, dx$		
$\int_0^\infty \frac{x}{1+x^2} \, dx$		



Poll

$$\int \frac{1}{2x-3}\,dx =$$

A $\dfrac{-2}{(2x-3)^2} + C$

B $\log(2x-3)/2 + C$

C $2\log(2x-3) + C$

D $\dfrac{-1}{(2x-3)^2} + C$

Idea: Algebra

The Method of Partial Fractions is more like Algebra than Calculus

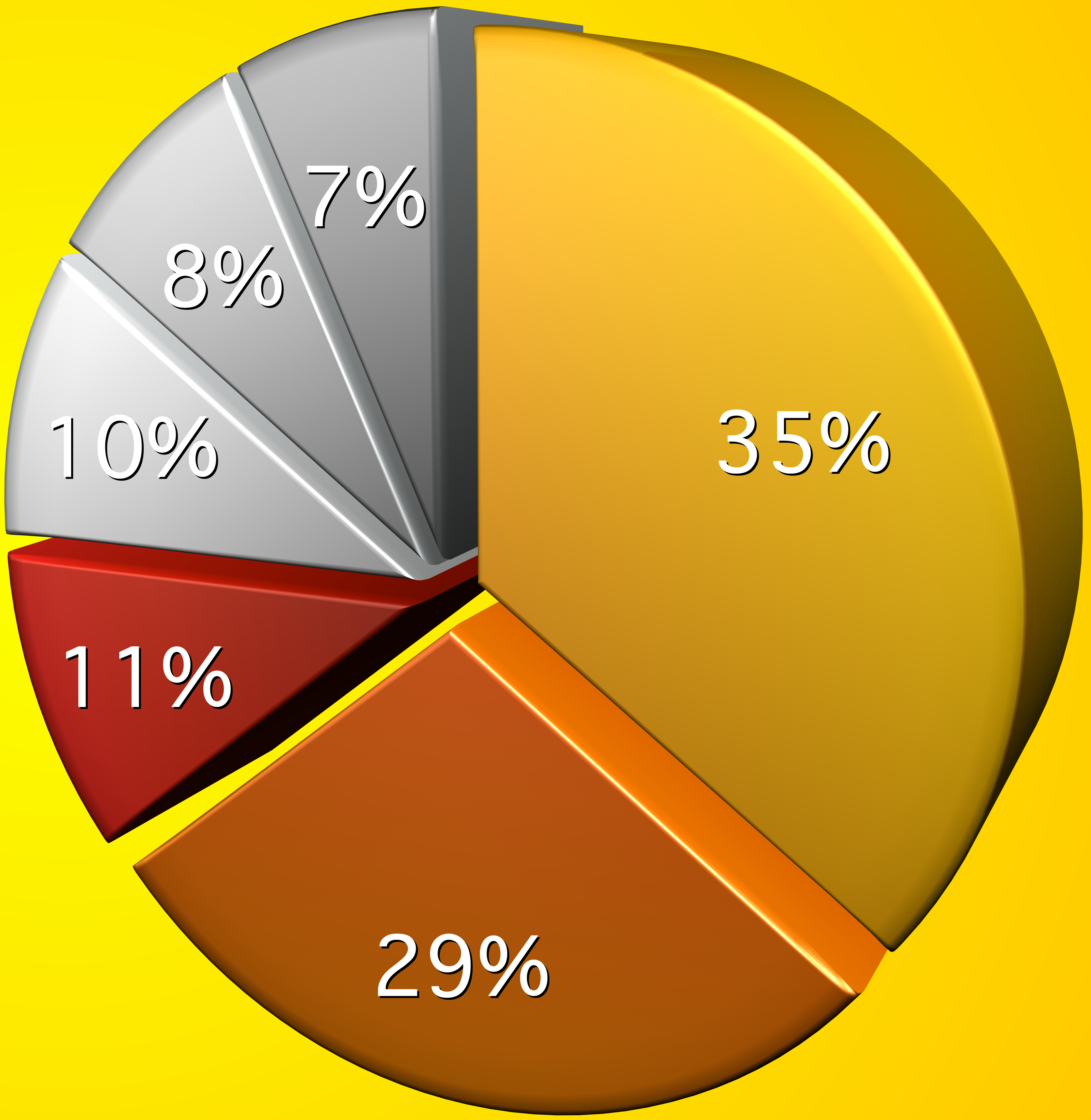
$$\frac{1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

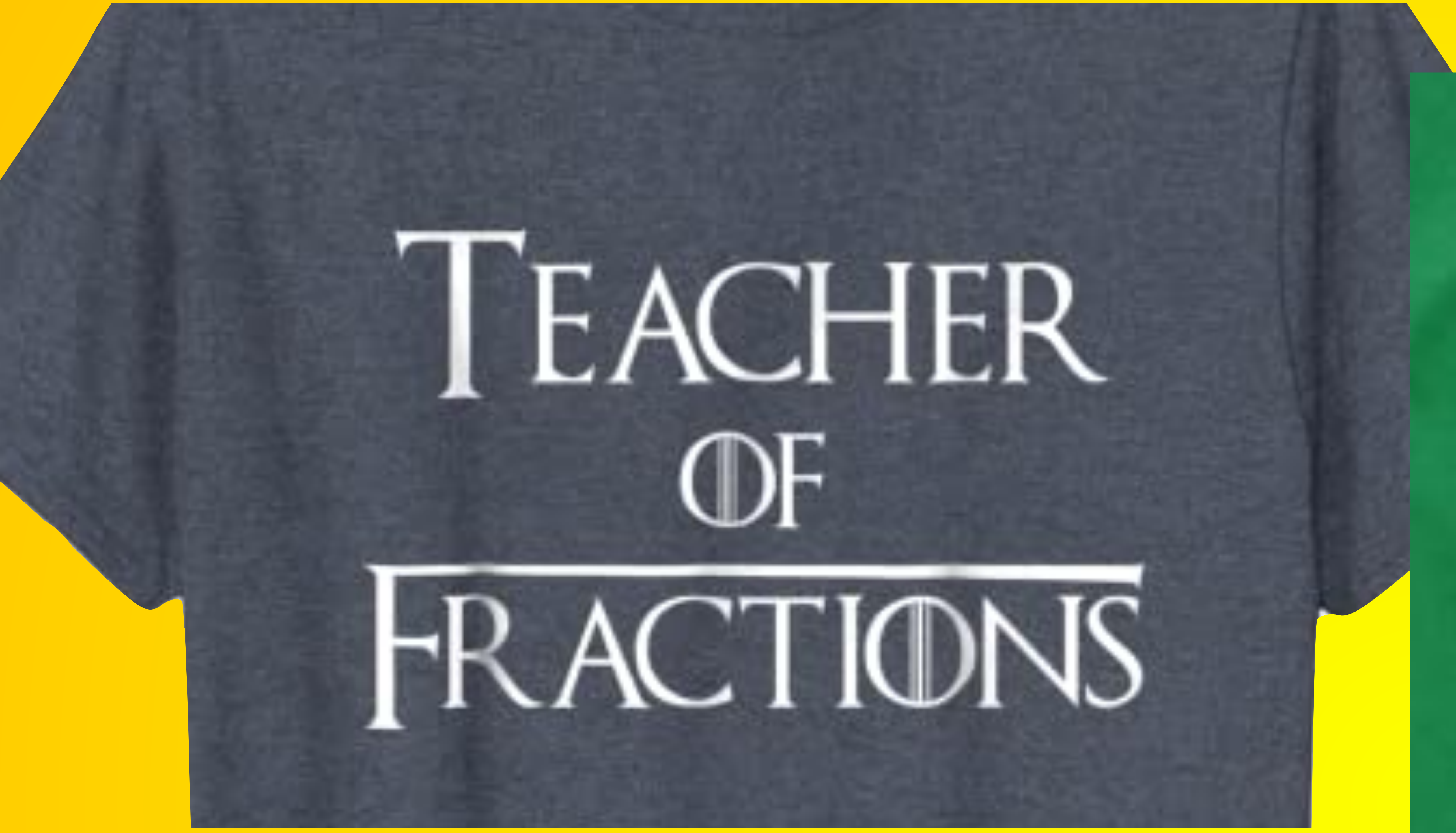
how to integrate?

this we know how to integrate

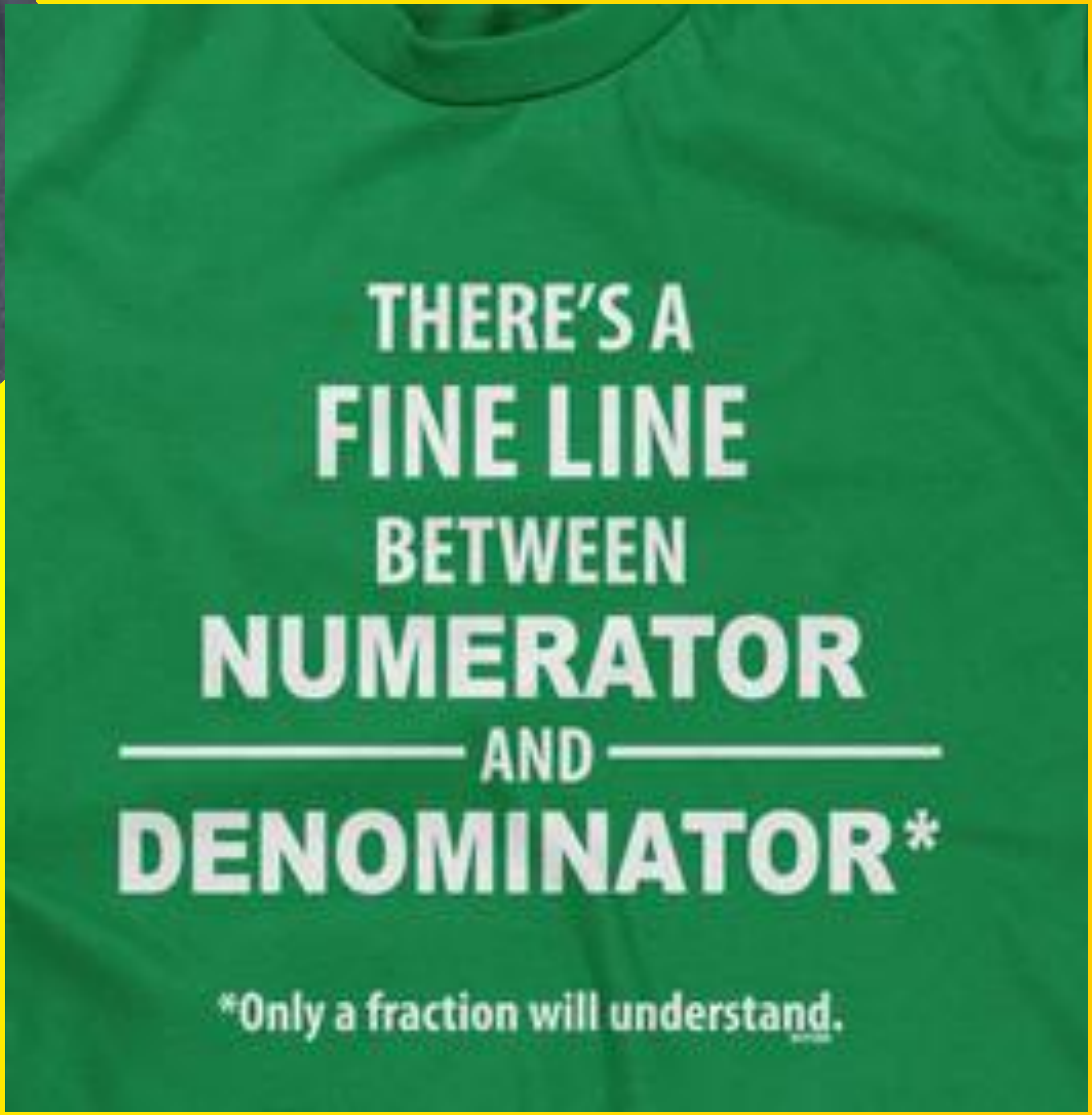
Fractions

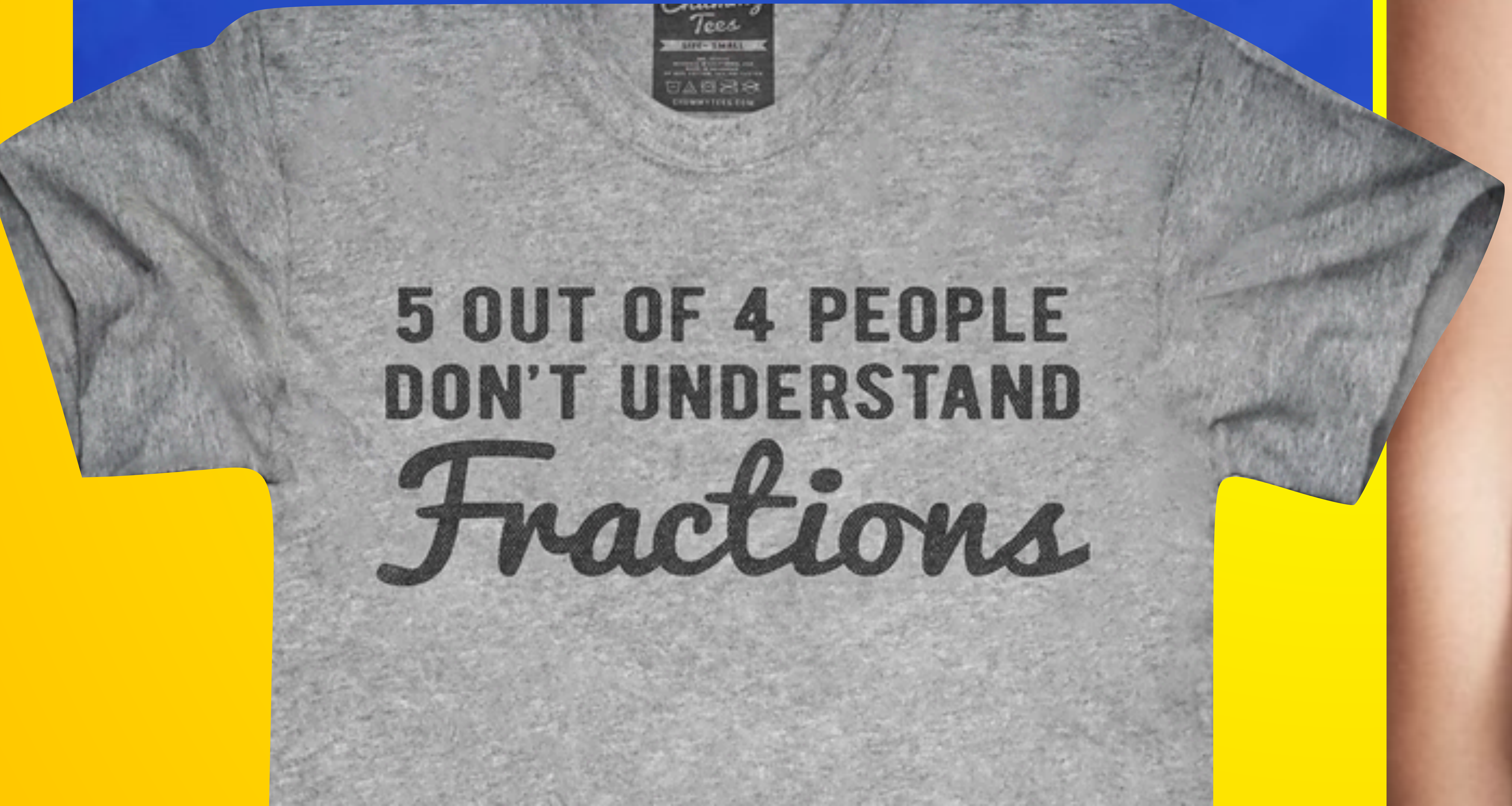
Are important!




TEACHER
OF
FRACTIONS


I
FRACTIONS


THERE'S A
FINE LINE
BETWEEN
NUMERATOR
AND
DENOMINATOR*
*Only a fraction will understand.


I'M IN BITS OVER
FRACTIONS

5 OUT OF 4 PEOPLE
DON'T UNDERSTAND
Fractions

You think you know
all about
FRACTIONS...
But
You don't know the
1/2 of it.

Prototype

$$\int \frac{1}{x^2 - 4}\, dx = ?$$

$$\frac{1}{x^2 - 4} = \frac{A}{x - 2} + \frac{B}{x + 2}$$

How do we find the constants A and B?

Comparing Coefficients

First Method

$$\frac{1}{x^2 - 4} = \frac{A}{x-2} + \frac{B}{x+2}$$

Make a common denominator

$$\frac{1}{x^2 - 4} = \frac{A(x+2)}{(x-2)(x+2)} + \frac{B(x-2)}{(x+2)(x-2)}$$

Comparing Coefficients

$$\frac{1}{x^2-4} = \frac{A(x+2)}{(x-2)(x+2)} + \frac{B(x-2)}{(x+2)(x-2)}$$

$$1 = A(x+2) + B(x-2)$$

$$\left| \begin{array}{l} 0 = A + B \\ 1 = (A - B)2 \end{array} \right|$$

system of equations

Solve for A,B

A=1/4, B=1/4

Residue Method

Second Method

$$\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

Multiply with (x-2), simplify, then put x=2.

$$\frac{1 \; (x-2)}{x^2-4} = \frac{A \; (x-2)}{x-2} + \frac{B \; (x-2)}{x+2}$$

$$\frac{1}{x+2} = A \qquad \frac{1}{2+2} = A$$

Residue Method

Now, multiply with (x+2), simplify, then put x=-2.

$$\frac{1}{x^2-4}(x+2) = \frac{A}{x-2}(x+2) + \frac{B}{x+2}(x+2)$$

$$\frac{1}{x-2} = B \qquad \frac{1}{-2-2} = B$$

Lets do this one

$$\frac{1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

```
f[x_] := 1/((x - 1) (x - 2) (x - 3));
Fit[Table[{x, f[x]}, {x, 4, 10}], {1/(x - 1), 1/(x - 2), 1/(x - 3)}, x]
```

Work

A $\displaystyle\int \frac{1}{(x-5)(x+3)}\,dx$

B $\displaystyle\int \frac{x}{(x-1)(x-2)(x-3)}\,dx$

C $\displaystyle\int \frac{1}{1-x^4}\,dx$

Jam

A

$$\int \frac{1}{(x-2)(x-7)}\,dx$$

B

$$\int \frac{1}{(x-5)(x-2)(x-3)}\,dx$$


On Youtube

Quit