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# Unit 26

## Partial fractions

Residue method

multiply

$$\frac{1(x-2)}{x^2-4} = \frac{A(x-2)}{x-2} + \frac{B(x-2)}{x+2}$$

simplify

$$\frac{1}{x+2} = A + \frac{B(x-2)}{x+2}$$

put  $x=2$

$$A = \frac{1}{4}$$

$$\frac{1(x+2)}{x^2-4} = \frac{A(x+2)}{x-2} + \frac{B(x+2)}{x+2}$$

$$\frac{1}{x-2} = \frac{A(x+2)}{x-2} + B$$

put  $x=-2$

$$-\frac{1}{4} = B$$

$$\int \frac{1}{x^2-4} dx = \frac{1}{4} \int \frac{1}{x-2} dx - \frac{1}{4} \int \frac{1}{x+2} dx$$

$$= \frac{1}{4} \log(x-2) - \frac{1}{4} \log(x+2) + C$$

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Traditional method

This is much simpler than the other method:

$$\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

common denominator

$$\frac{1}{x^2-4} = \frac{A(x+2) + B(x-2)}{x^2-4}$$

$$1 = A(x+2) + B(x-2)$$

$$0 \cdot x + 1 = (A+B)x + (A-B) \cdot 2$$

compare coefficients

$$\begin{cases} 1 = (A-B) \cdot 2 \\ 0 = A+B \end{cases}$$

system of equations

$$\begin{cases} 1 = 2A - 2B \\ 0 = 2A + 2B \end{cases}$$

How to get A, B?  $B = -\frac{1}{4}$

( $2A = -2B \rightarrow$  plug in)

$$A = \frac{1}{4}$$

add up the equations

$$1 = 4A + 0$$

$$\textcircled{2} \int \frac{1}{(x-1)(x-2)(x-3)} dx$$

$$\frac{1}{\cancel{(x-1)}(x-2)(x-3)} = \frac{\textcircled{A}}{\cancel{x-1}} + \frac{B}{x-2} + \frac{C}{x-3}$$

$x=1$       ① multiply      ② simplify

$$\frac{1}{(1-2)(1-3)} = A$$

$$\frac{1}{2}$$

$$\frac{1}{(x-1)\cancel{(x-2)}(x-3)} = \frac{A}{x-1} + \frac{B}{\cancel{x-2}} + \frac{C}{x-3}$$

$x \rightarrow 2$

$$\frac{1}{(2-1)(2-3)} = -1$$

$$B = -1$$

$$\frac{1}{(x-1)(x-2)\cancel{(x-3)}} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{\cancel{x-3}}$$

$x \rightarrow 3$

$$\frac{1}{2} = \frac{1}{2}$$

$$C = \frac{1}{2}$$

$$\int \frac{1}{(x-1)(x-2)(x-3)} dx$$

The answer to the problem.

$$\frac{1}{2} \log(x-1) - \log(x-2) + \frac{1}{2} \log(x-3) + C$$

## work

(A)  $\int \frac{1}{(x-5)(x+3)} dx$

(B)  $\int \frac{x}{(x-1)(x-2)(x-3)} dx$

(C)  $\int \frac{1}{1-x} dx$

(A)  $\frac{1}{(x-5)(x+3)} = \frac{A}{x-5} + \frac{B}{x+3}$

$\frac{1}{8} = A$

$$\frac{1}{(x-5)(x+3)} = \frac{A}{x-5} + \frac{B}{x+3}$$

$$= \frac{-1}{8} = B$$

(B)

$$\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$x=1 \quad \frac{1}{(1-2)(1-3)} = \frac{1}{2} = A$$

$$\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$x=2$$

$$\frac{2}{(2-1)(2-3)} = -2 = B$$

$$\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$x=3$$

$$\frac{3}{(3-1)(3-2)} = C$$

$$\frac{3}{2} = C$$

$$\textcircled{c} \quad \frac{1}{x^2-4} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$$

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